

Scanning Inflation

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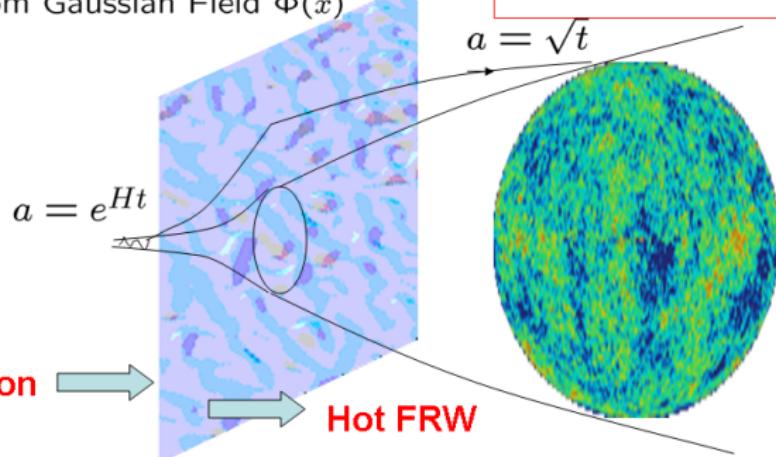
Introduction

- Top-down approach to inflation
seeks to embed it in a fundamental theory
- Bottom-up approach to inflation
reconstruction of acceleration trajectories

Scalar metric Fluctuations from Inflation
 $ds^2 = -(1 - 2\Phi)dt^2 + (1 + 2\Phi)a^2d\vec{x}^2$

Initial conditions from Inflation →

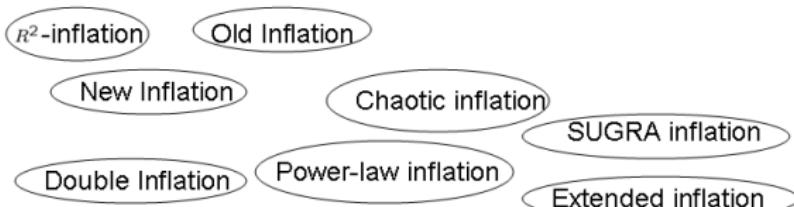
Random Gaussian Field $\Phi(\vec{x})$



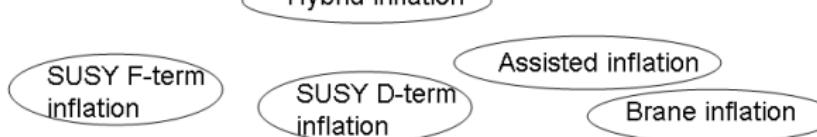
$$\begin{aligned}\Omega_{tot} &= 1 \\ k^3 \Phi_k^2 &\rightarrow P_s = A_s k^{n_s - 1} \\ P_T &= \frac{H^2}{M_p^2} k^{n_T} \\ N &= 62 - \ln \frac{10^{16} GeV}{V_{inf}^{1/4}}\end{aligned}$$

Inflation in the context of ever changing fundamental theory

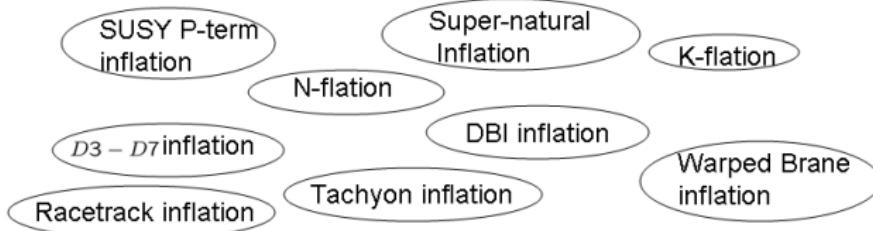
1980



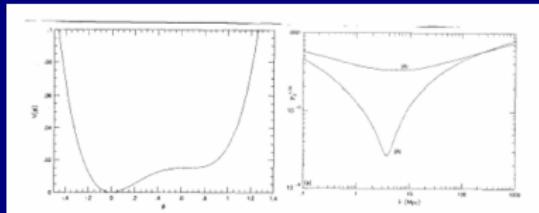
1990



2000



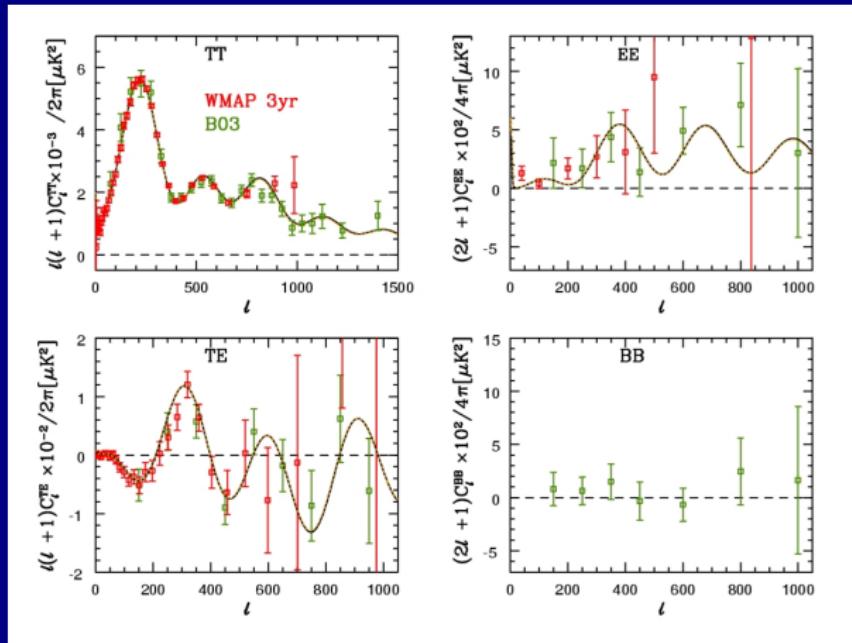
$$V = m^2 \phi^2 + \lambda_1 \phi^3 + \lambda_2 \phi^4$$



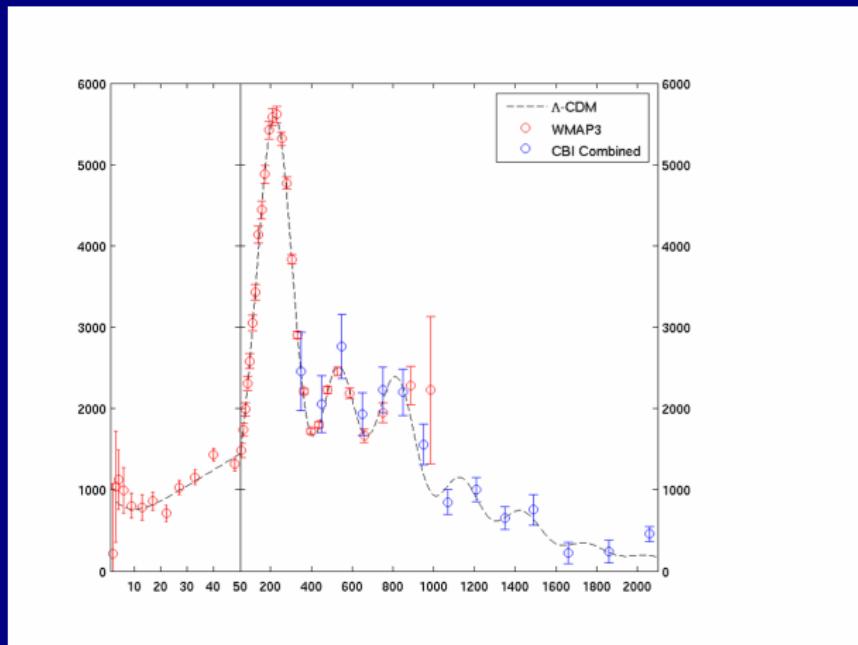
- renormalizable
- different choices of parameters give quite different powerspectra
- but also different shapes of the potential:
 - exponential
 - SUGRA
 - ...



BOOMERANG sees 4th peak



CBI sees 5th peak

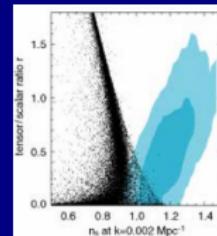


Slow-Roll Parameters

- Equations of Motion

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0$$

$$\frac{8\pi}{3m_p^2} \left(\frac{1}{2}\dot{\phi}^2 + V \right) = H^2$$

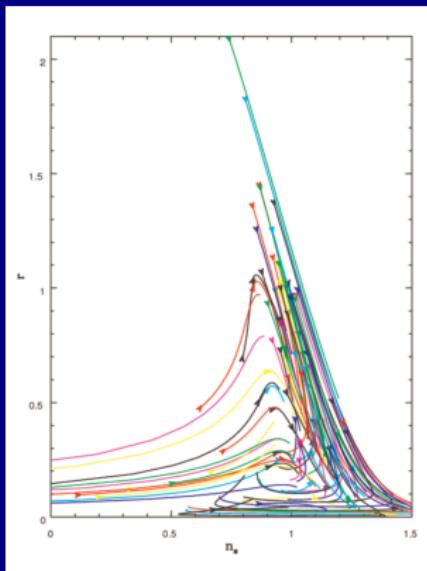


- Slow roll parameters

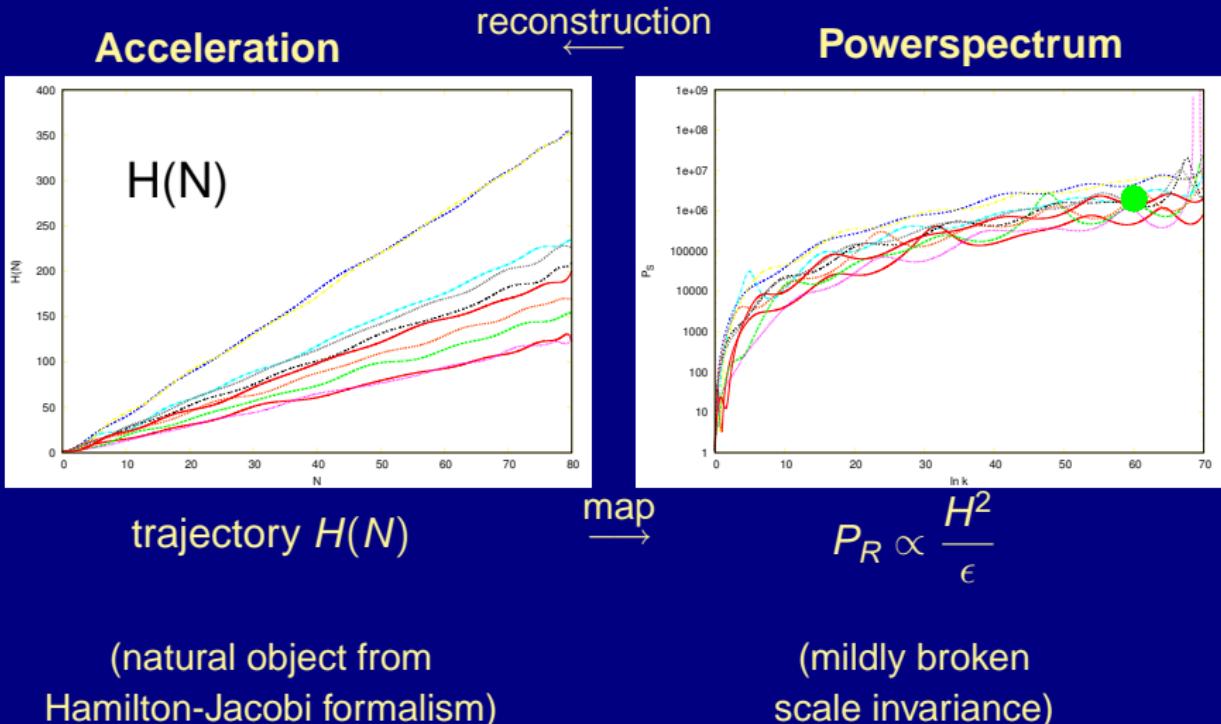
$$\epsilon = -\frac{\dot{H}}{H^2}, \quad \eta = \frac{m_p^2}{4\pi} \frac{H''}{H}, \quad \zeta^2 = \left(\frac{m_p^2}{4\pi} \right)^2 \frac{H'''H'}{H^2}, \quad \sigma = 2\eta - 4\epsilon$$

- Flow Equations
- Powerspectra

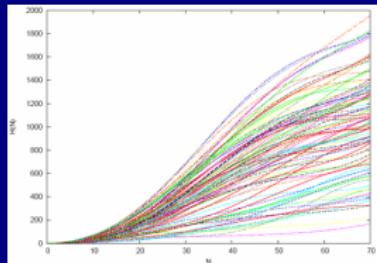
$$P_R \equiv A_S \left(\frac{k}{k_{\text{pivot}}} \right)^{n_s-1} \propto \frac{H^2}{\epsilon}, \quad P_G \equiv A_t \left(\frac{k}{k_{\text{pivot}}} \right)^{n_t} \propto H^2$$



varying N moves the points in observable space around

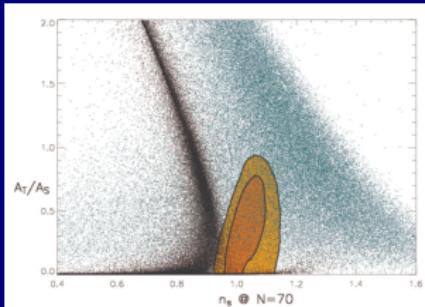
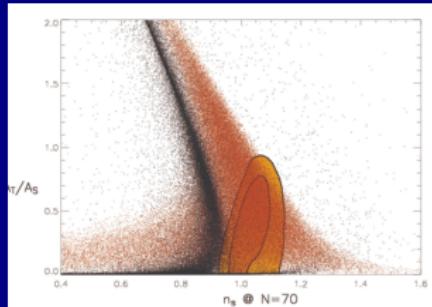


Ensemble of Inflationary Trajectories



various trajectories and time variables

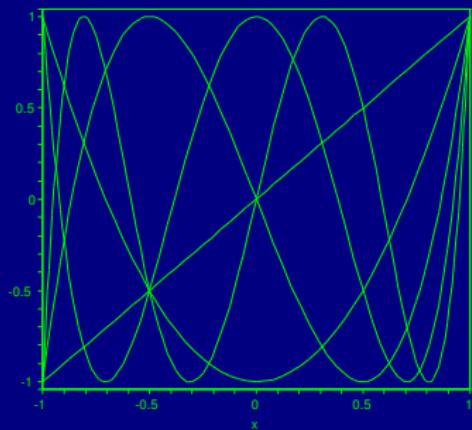
- $\epsilon, H, \ln(P_R), \ln(P_G)$
- $N, \ln(k)$
- ...



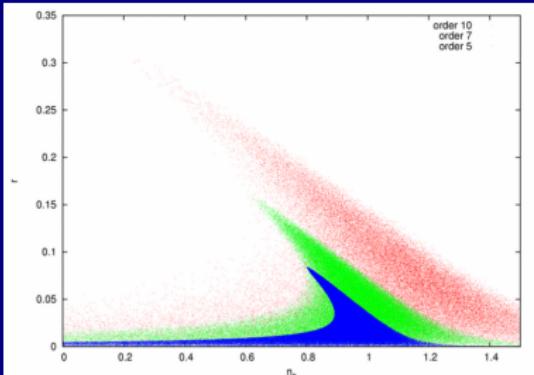
⇒ space opens up, fast

Trajectories

- N : # of efolds $dN = -Hdt$
- Constraints during inflation
 - $0 \leq \epsilon \leq 1$
 - $H > 0$
- at the end of inflation $\epsilon = 1$
- Expansion to arbitrary order
 - $H(N) = \sum_i c_i T_i(x)$
 - with $x = \frac{2N - N_{\max}}{N_{\max}}$, Chebyshev polynomials $T_i(x)$
(uniformly best approximation to “true” function)

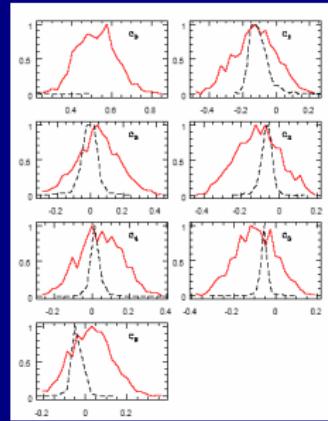


$$T_n(\cos(x)) = \cos(nx)$$

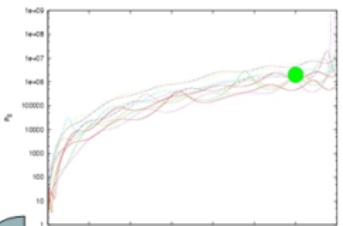


⇐ Space opens more with higher order polynomials

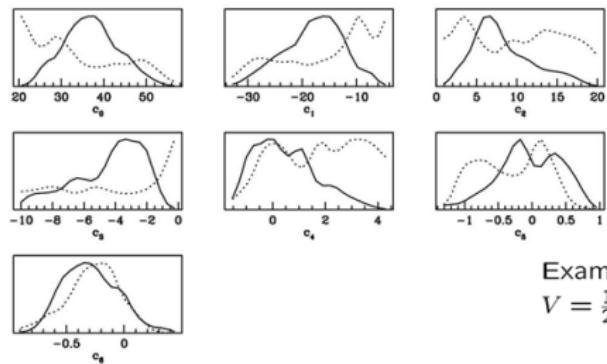
- red c_n
our method
- black c_n
Chebyshev-transformed flow equations



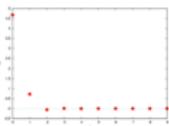
Observational constraints on trajectories



Markov Chain Monte Carlo



Example of c_n for
 $V = \frac{1}{2}m^2\phi^2$



Nodal points

- choose $f(x_i)$ at nodal points x_i

- window function $w_j(x)$:

$$f(x_i) = \sum_j c_j T_j(x_i) \Rightarrow c_j = T_j(x_i)^{-1} f(x_i)$$

$$f(x) = \sum_i c_i T_i(x) = \sum_i f(x_i) \underbrace{\sum_j T_j(x_i)^{-1} T_j(x)}_{w_i}$$

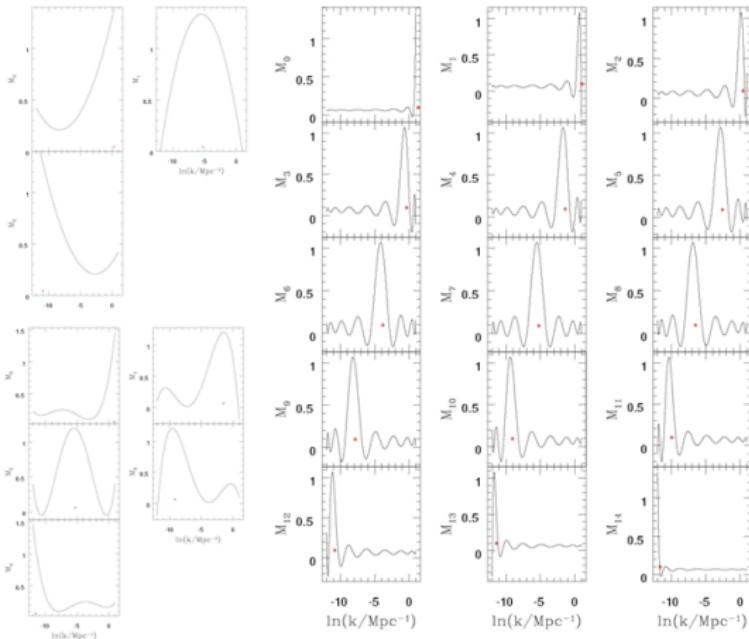
- $f(x) = H, \ln(H), \epsilon, \ln(\epsilon), P_R, \ln(P_R), P_G, \ln(P_G), \dots$
- $x = N, \ln(k)$

$$\Rightarrow \epsilon \in [0, 1]$$

Chebyshev nodal modes: order 3, 5, 15 (Fourier at high order)

Chebyshev modes are liner combination

How far does choice of values at nodal points "feel" out?



Displaying
Trajectory
constraints:

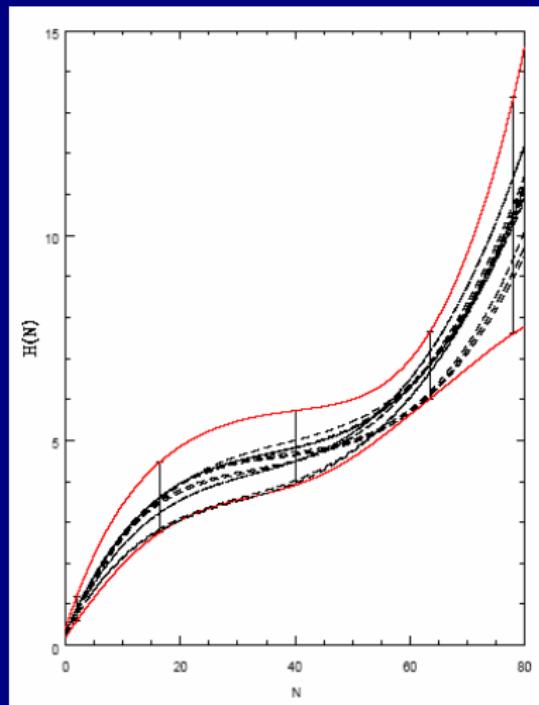
If Gaussian likelihood,
compute χ^2
where 68%
probability, and
follow the ordered
trajectories to

In $L/L_m = -\chi^2/2$,
displaying a
uniformly sampled
subset.

Errors at nodal
points in trajectory
coefficients can
also be displayed.

time variable $N \leftrightarrow \ln k$

- $N: N \in [0, 70]$
but: only interested in
observable interval
 $\Delta N \approx 5$
- $\ln k: \text{observable } k \in [10^{-4} \text{Mpc}^{-1}, 1 \text{Mpc}^{-1}]$

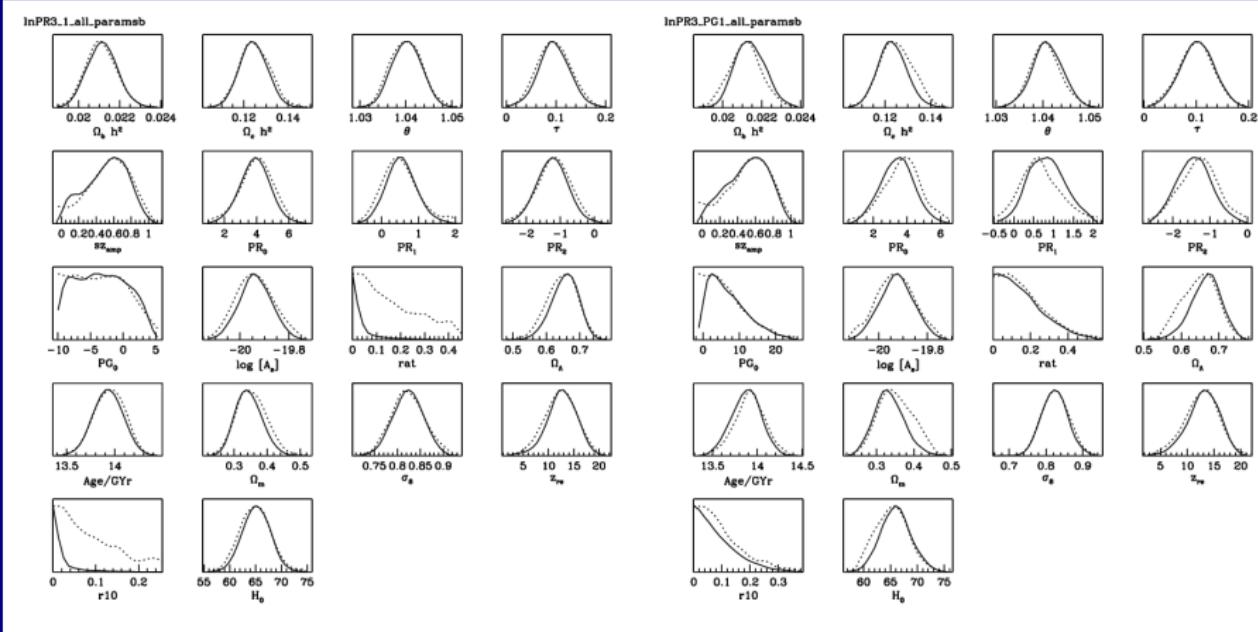


- Expansion of $\ln(P_R(\ln k))$ to order 3, $\ln(P_G(\ln k))$ to order 1
- Expansion of $\ln(P_R(\ln k))$ to order 3, $P_G(\ln k)$ to order 1
- Expansion of $\epsilon(\ln k)$ to order 10, keeping outmost points fixed

Datasets:

- ACBAR
- BOOMERANG
- CBI
- DASI
- MAXIMA
- VSA
- WMAP
- 2dF
- SDSS

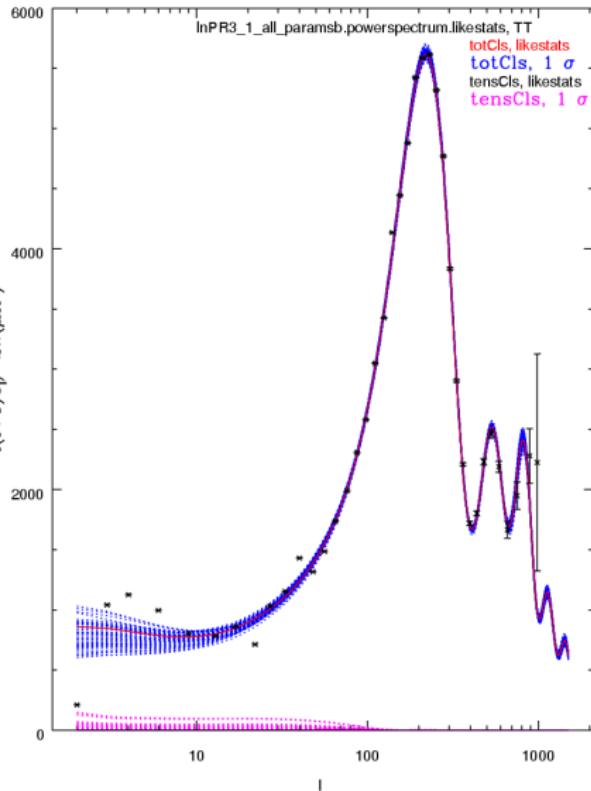
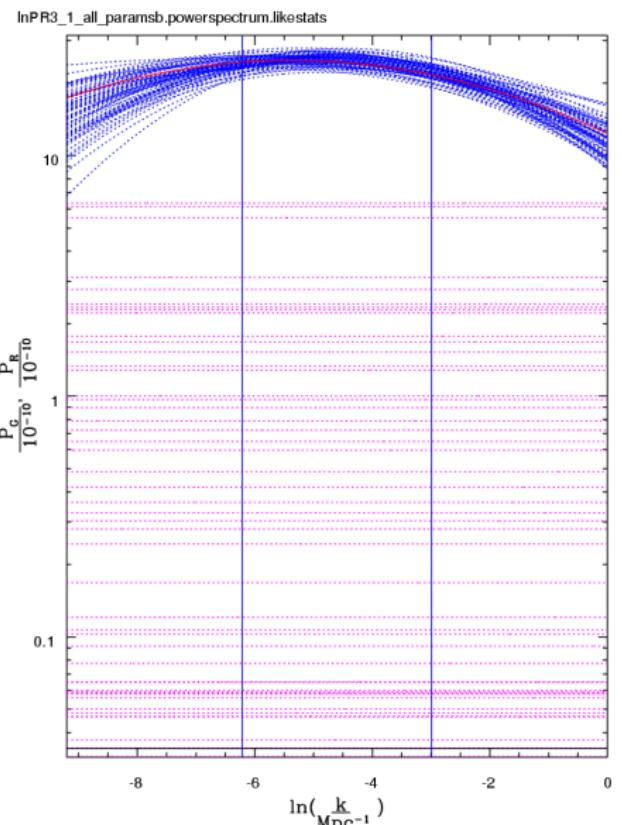
Introduction
 CMB experiments
Bottom Up Approach
 Conclusions



cosmological parameters:

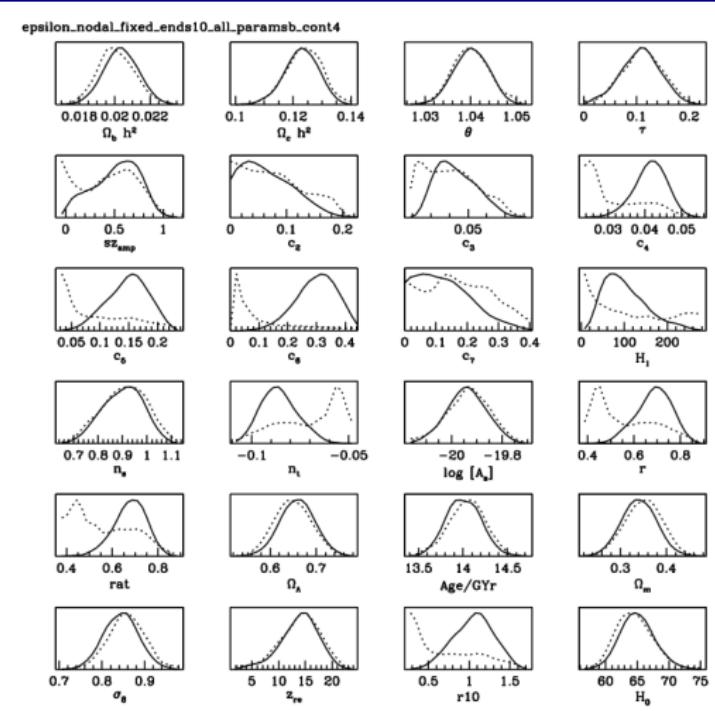
parameter	WMAP3	InPR 3	InPG 1	InPR3 PG1
$\frac{A}{10^{-10}}$	21^{+1}_{-2}	22^{+1}_{-2}	22^{+2}_{-1}	
$n_s(0.002 Mpc^{-1})$	$1.21^{+0.1}_{-0.15}$	1.16		1.22
$n_s(0.05 Mpc^{-1})$	$0.845^{+0.036}_{-0.044}$	0.78		0.79
$\frac{dn_s}{d \ln k}$	$-0.112^{+0.1}_{-0.15}$	-0.117		-0.13
τ	$0.097^{+0.03}_{-0.036}$	$0.096^{+0.03}_{-0.03}$	$0.102^{+0.03}_{-0.03}$	
...

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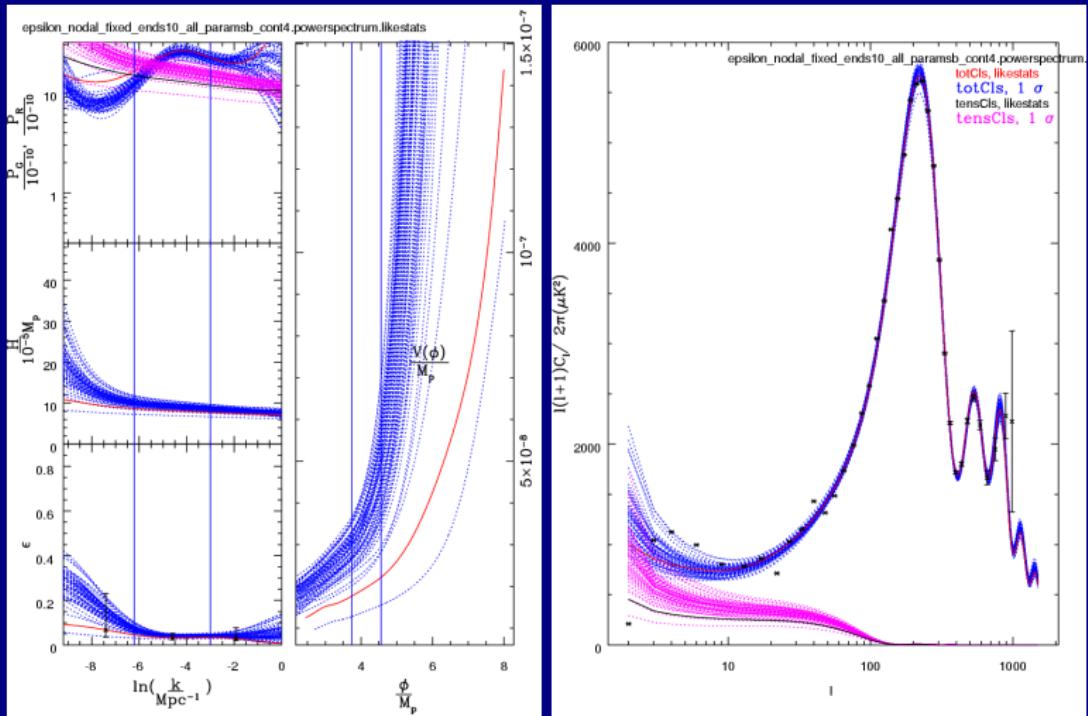
Expansion of

- $\ln(P_R) = \sum_i PR_i T_i(x) \times 10^{-10}$
- $\ln(P_G) = \frac{1}{2} PG_0 T_0(x) \times 10^{-10}$
 \Rightarrow for $PG_0 \in [-10, 5]$, P_G is preferably exponentially small
- $P_G = \frac{1}{2} PG_0$
 \Rightarrow for $PG_0 \in [0, 5]$, P_G is uniform

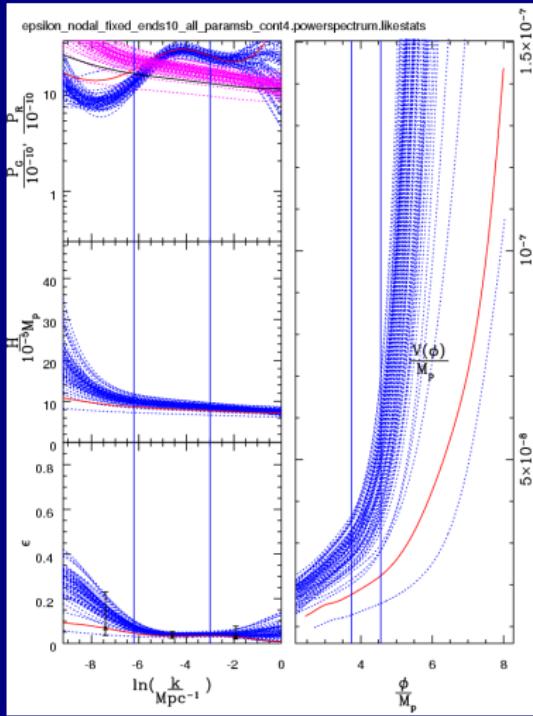
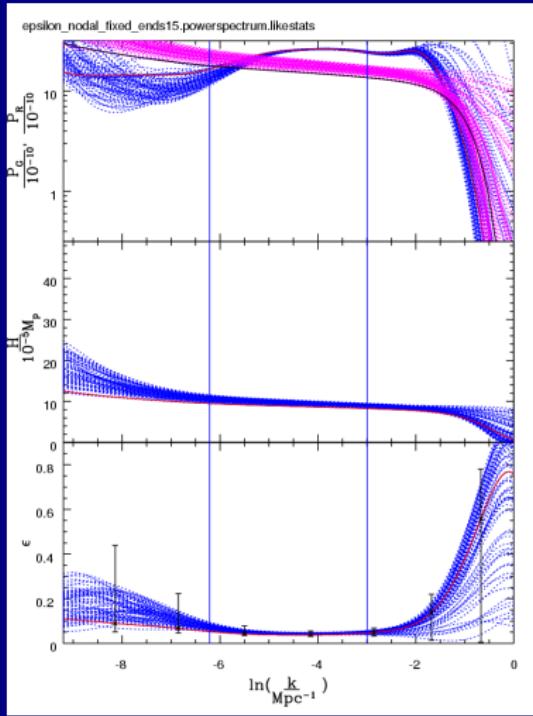


large tensors!
(compatible with
WMAP3 if run-
ning of n_s is al-
lowed)

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WMAP1 \leftrightarrow WMAP3

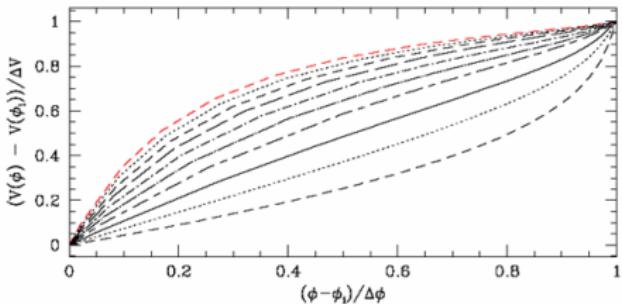


Degeneracy of the Potential Reconstruction

known $P_s(k) \rightarrow$ reconstruct $V(\phi)$

$$P_s(k) = \frac{8\pi H^4}{M_p^4 H'^2}$$

$$V(\phi) = \frac{M_p^4}{32\pi^2} \left(\frac{12\pi}{M_p^2} H^2 - H'^2 \right)$$



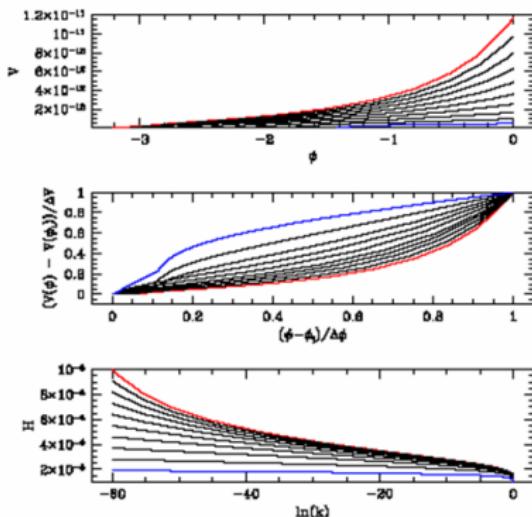
$$\phi - \phi_0 = \frac{M_p^2}{2} \int_{\ln k_0}^{\ln k} d \ln k' \frac{\sqrt{P_s(k')}}{H^2(k')} \frac{dH}{d \ln k'}$$

$$\frac{dH}{d \log k} = \frac{H^3}{H^2 - \pi M_p^2 P_s(k)}$$

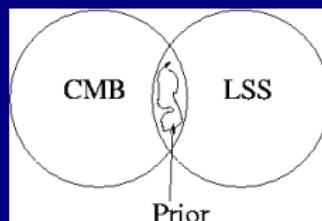
Degeneracy is lifted by fixing $P_{GW} = \frac{H_*^2}{M_p^2}$

Example $P_s(k) = k^{n_s - 1}$

$n_s = 0.98$



Why large tensor modes?



- CMB: $\frac{\Delta T}{T}$ sensitive to $P_R + P_G$
⇒ Favors one set of trajectories
 - LSS: $\frac{\Delta T}{T}$ sensitive to P_R only
⇒ Favors another set of trajectories
 - Prior ⇒ Weighs different trajectories differently
- ⇒ Combination of these effects picks a set of trajectories

Can do P_R , P_G plots for specific potentials:
solve

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0$$

$$H^2 = \frac{1}{3M_p^2} \left(\frac{1}{2}\dot{\phi}^2 + V(\phi) \right)$$

obtain: $H(t)$, $\epsilon(t)$, $a(t)$

$P_R(\ln k)$ from parametric plot $[\ln(aH), \frac{H^2(t)}{\epsilon(t)}]$

$P_G(\ln k)$ from parametric plot $[\ln(aH), H^2(t)]$

but: proper normalization of $a(t)$ depends on details of
(p)reheating

⇒ shift powerspectrum to left/ right

Conclusions

- Classical Observables \leftrightarrow power spectra
- Increasing the order of Chebyshevization opens the space of classical observables
- Reconstruction of inflationary trajectories
- Priors
- Degeneracy
- LSS \leftrightarrow CMB