Trajectories in Multi Kähler Inflation

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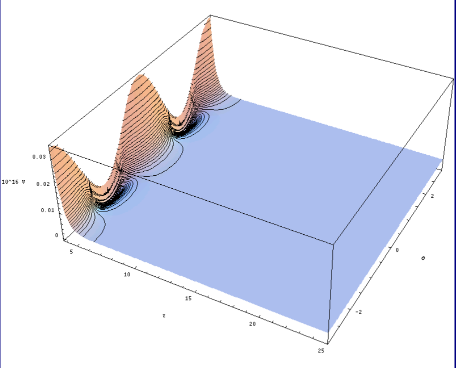
Introduction



- $W = W_0 + \sum_i A_i e^{-a_i T_i}, \; T_i = \tau_i + i\theta_i$
- $K = -2\ln(\mathcal{V} + \frac{\xi}{2}), \; \mathcal{V} = \alpha \left(\tau_1^{3/2} - \sum_{i=2}^{n} \lambda_i \tau_i^{3/2} \right)$

SUGRA

$$V = e^K \left(K^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - 3|W|^2 \right)$$

- $D_i = \partial_i + K_i,\ K_i = \frac{\partial K}{\partial T^i},\ K_{i\bar{j}} = \frac{\partial^2 K}{\partial T^i \partial \bar{T}^{\bar{j}}}$
- pen and paper
- # terms $\propto (\#\text{fields})^2$
- $\Rightarrow$ Mathematica: $\approx$ 1 min for 3 fields
- Here: 3 complex fields $(T_1, T_2) \Leftrightarrow (\mathcal{V}, T_2)$

⇒ Resulting potential

$$V(T_1, ..., T_n) =$$

$$\frac{12 W_0^2 \xi}{(4\mathcal{V}-\xi)(2\mathcal{V}+\xi)^2} + \sum_{i=2}^{n} \frac{12 e^{-2a_i\tau_i} \xi A_i^2}{(4\mathcal{V}-\xi)(2\mathcal{V}+\xi)^2} + \frac{16 (a_i A_i)^2 \sqrt{\tau_i} e^{-2a_i\tau_i}}{3\alpha\lambda_2 (2\mathcal{V}+\xi)}$$

$$+ \frac{32 e^{-2a_i\tau_i} a_i A_i^2 \tau_i (1+a_i\tau_i)}{(4\mathcal{V}-\xi)(2\mathcal{V}+\xi)} + \frac{8 W_0 A_i e^{-a_i\tau_i} \cos(a_i\theta_i)}{(4\mathcal{V}-\xi)(2\mathcal{V}+\xi)} \left(\frac{3\xi}{(2\mathcal{V}+\xi)} + 4a_i\tau_i \right)$$

$$+ \sum_{\substack{i,j=2 \\ i<j}}^{n} \frac{A_i A_j \cos(a_i\theta_i - a_j\theta_j)}{(4\mathcal{V}-\xi)(2\mathcal{V}+\xi)^2} e^{-(a_i\tau_i + a_j\tau_j)} \left[32(2\mathcal{V}+\xi)(a_i\tau_i + a_j\tau_j \right.$$

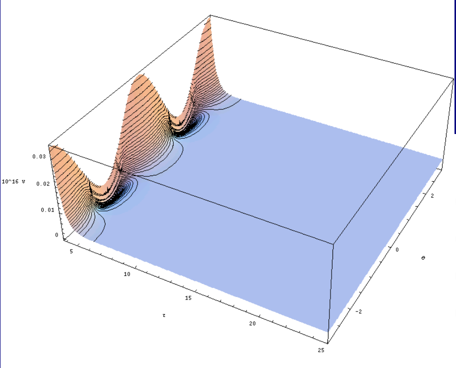
$$\left. + 2a_i a_j \tau_i \tau_j) + 24\xi \right]$$

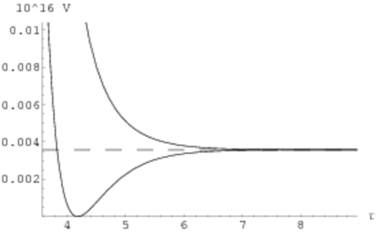
Stabilization and Uplift

- inflaton T_2
- $\mathcal{V}$ stabilized by extra fields $T_3 \ldots T_n$: $\sum_{i=2}^{n} \frac{\lambda_i}{a_i^{3/2}} \gg \frac{\lambda_2}{a_2^{3/2}}$
- assume $V = V(\mathcal{V}, T_3, \beta)$
- $V_{\mathrm{m}} < 0$
  - Need uplift $V_{\mathrm{up}} = \frac{\beta}{\mathcal{V}^\alpha}$
  - self-consistent uplift procedure:
    solve $\left\{ \begin{array}{c} V(\mathcal{V}, T_3, \beta) = \min \\ V(\mathcal{V}, T_3, \beta) = 0 \end{array} \right\} \Rightarrow V(T_2) \equiv V(\mathcal{V}_m, T_{3,m}, \beta, T_2)$
- inflaton $T_2 \equiv \tau + i\theta$

$$V(\tau, \theta) = \frac{8(a_2 A_2)^2 \sqrt{\tau} e^{-2a_2\tau}}{3\alpha\lambda_2\mathcal{V}_m} - \frac{4 W_0 a_2 A_2 \tau e^{-a_2\tau} \cos(a_2\theta)}{\mathcal{V}_m^2} + \Delta V$$

Form of the potential



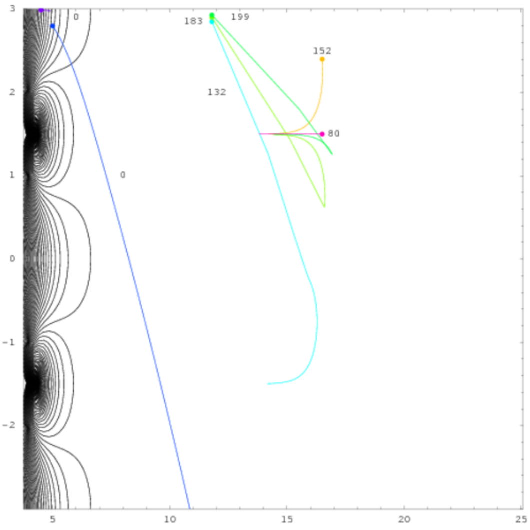


Equations of Motion

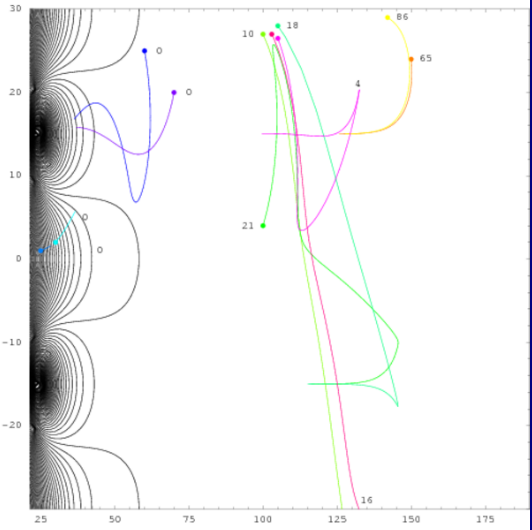
Parameter	W_0	a_2	A_2	λ_2	α	ξ	g_s	$\mathcal{V}$
Parameter set 1	300	$\frac{2\pi}{3}$	0.1	1	$1/9\sqrt{2}$	0.5	$1/10$	10^6
Parameter set 2	6×10^4	$\frac{2\pi}{30}$	0.1	1	$1/9\sqrt{2}$	0.5	$1/10$	10^8
Parameter set 3	4×10^5	$\frac{\pi}{100}$	1	1	$1/9\sqrt{2}$	0.5	$1/10$	10^9

Solve until $\epsilon = 1$:

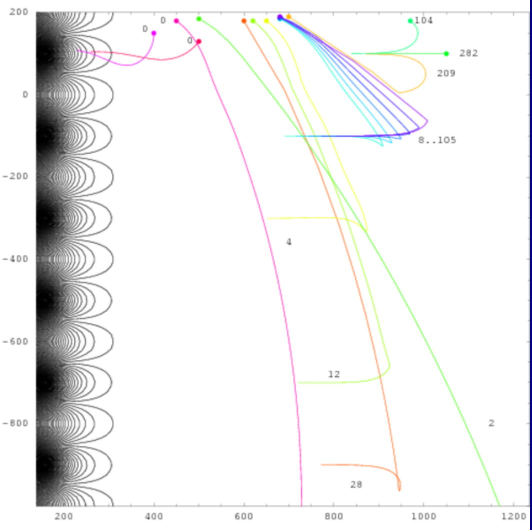
$$\begin{aligned}
\dot{\phi}^i &= \frac{1}{2a^3} G^{ij} P_j, \\
\dot{P}_i &= -\frac{1}{4a^3} \frac{\partial G^{kl}}{\partial \phi^i} P_k P_l - a^3 \frac{\partial V}{\partial \phi^i}, \\
\dot{a} &= aH, \\
\dot{H} &= -\frac{1}{4a^3} G^{ij} P_i P_j,
\end{aligned}$$



3
2
1
0
−1
−2
5
10
15
20
25
0
0
183
199
152
132
80



30
20
10
0
-10
-20
25
50
75
100
125
150
175
0
0
0
0
10
18
86
65
4
21
16



200
0
-200
-400
-600
-800
200
400
600
800
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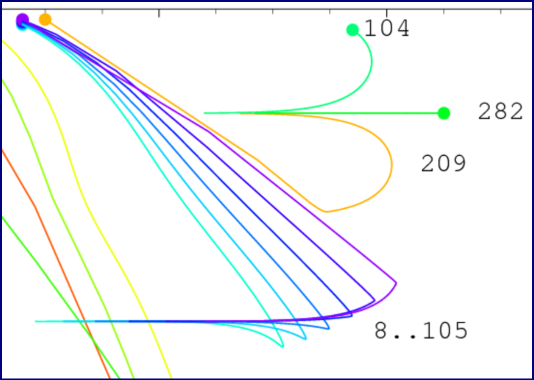
1200
0
0
0
104
282
209
8..105
4
12
2
28


104
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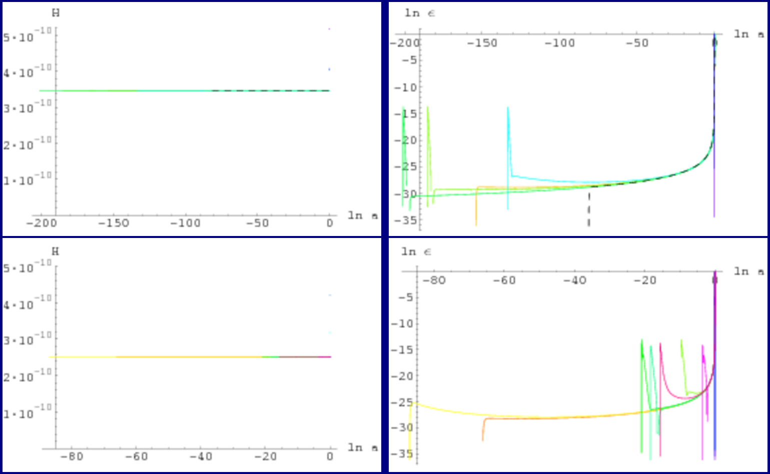
8..105

Features

- axionic direction
- dependence on initial conditions
- bifurcation points
- lots of efolds
- isocurvature perturbations
- $N = 40 \ldots 50$

(from $N(k) = 62 - \ln \frac{k}{6.96 \times 10^{-5}\,\mathrm{Mpc}^{-1}} + \Delta$, with $\Delta = -\ln \frac{10^{16}\mathrm{GeV}}{V_k^{1/4}} + \frac{1}{4} \ln \frac{V_k}{V_{\mathrm{end}}} - \frac{1}{3} \ln \frac{V_{\mathrm{end}}^{1/4}}{\rho_{\mathrm{reh}}^{1/4}}$)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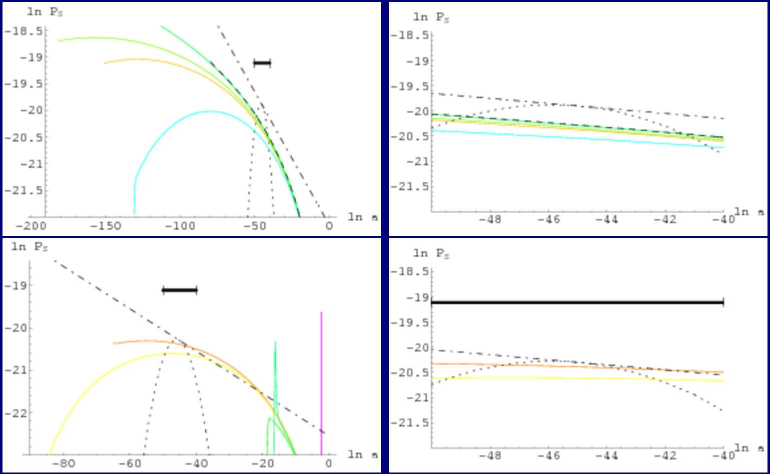
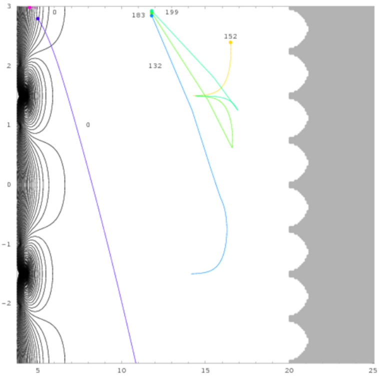


Template $P_S \propto k^{n_s - 1}$ with a) dash-dot: $n_s = 0.95$, $n_{\rm run} = 0$
b) dotted: $n_s = 0.95$, $n_{\rm run} = -0.055$, pivot point $N = 45$

Initial conditions

- Stabilization from 3rd … n^{th} field $T_3 \dots T_n$
- $\Rightarrow$ uniform (?) distribution of initial values of (τ, θ)
- maybe existence of region of eternal inflation? $\Rightarrow$ YES!



Conclusions

- rich structure
  - axionic direction
  - bifurcation points
  - lots of efolds
- stochastic regime