Inflation then and now: How can we reliably determine whether dark energy is given by a cosmological constant or whether the power spectrum of scalar fluctuations is exactly scale invariant?

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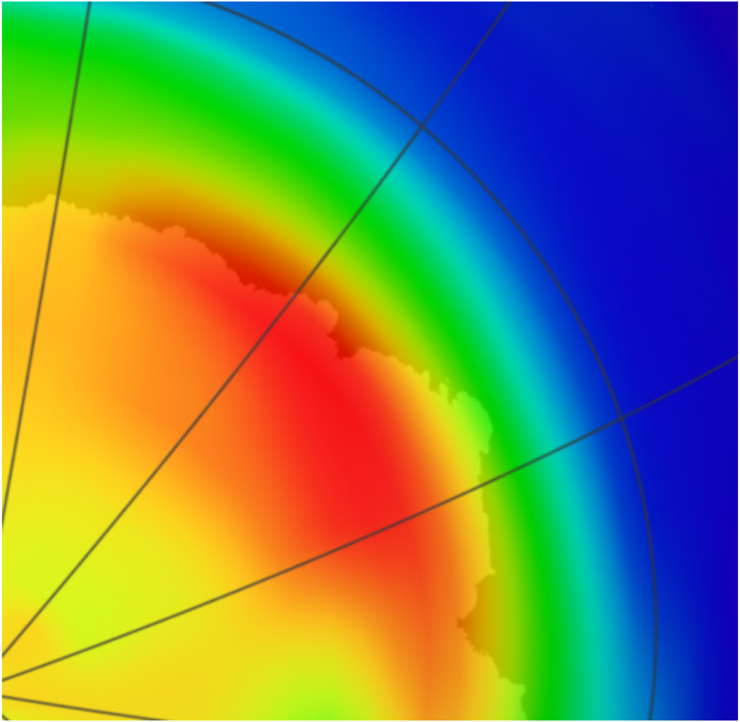
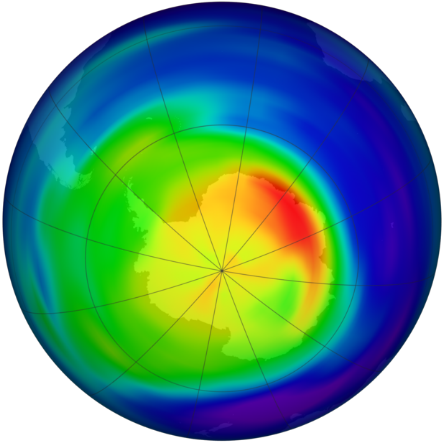
G. Starkman, R. Trotta, PMV, `arXiv:0811.2415`
G. Starkman, M. March, R. Trotta, PMV, `arXiv:0901.XXXX`

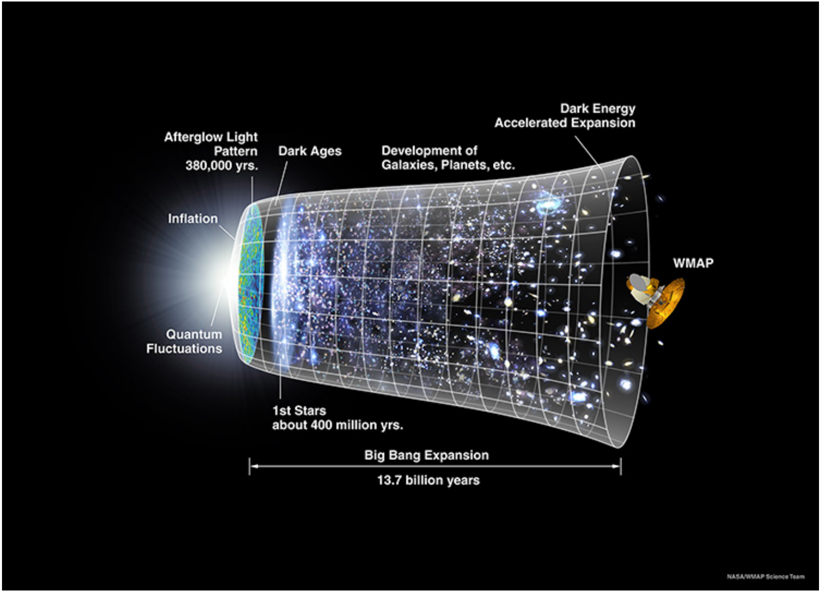




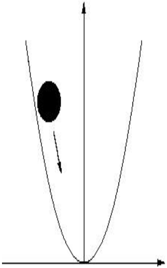
(Microwave Limb Sounder, NASA)

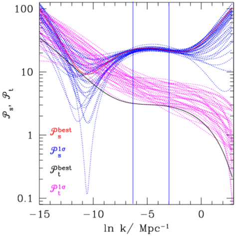
Outline

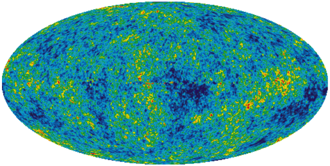
Early and late time inflation



(Early time) inflation

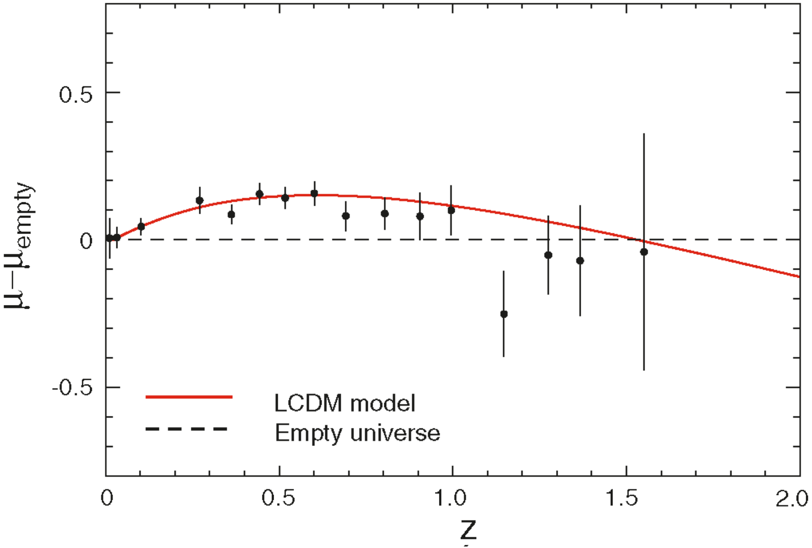






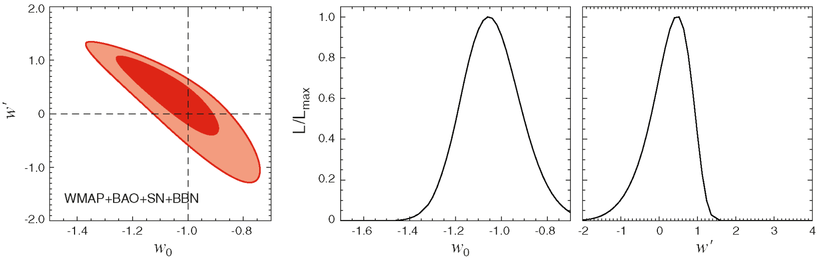
$$\mathcal{P}_s = \frac{k^3}{2\pi^2}\langle \mathcal{R}\mathcal{R} \rangle = \frac{1}{8\pi^2}\frac{H^2}{\epsilon {M_{\mathtt{pl}}}^2} = A_s k^{n_s - 1}$$

Late time inflation/ Dark energy



(WMAP5, Dunkley et al.)

Late time inflation/ Dark energy

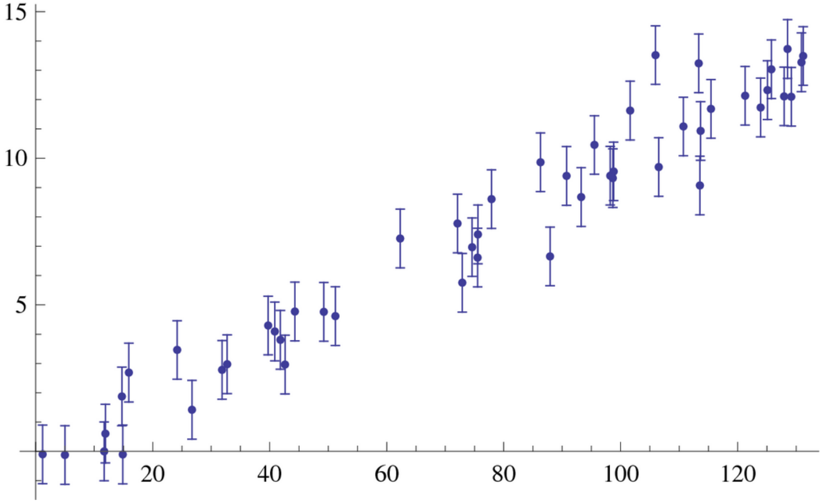


(WMAP5, Komatsu et al.)

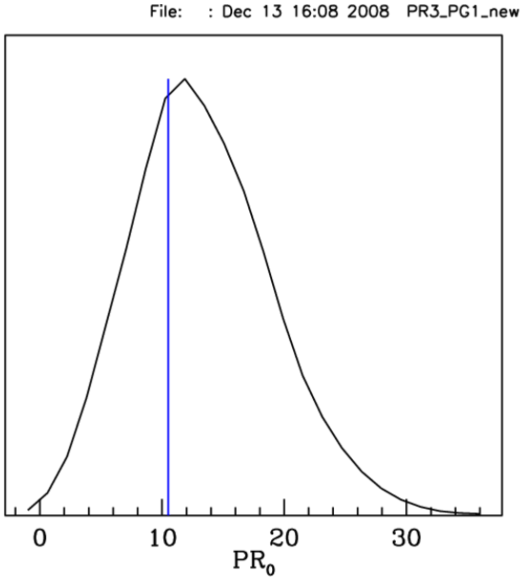
Key problems

- exactly flat power spectrum from inflation?
- dark energy is cosmological constant?

How to detect features?



- visual inspection
- χ^2/dof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Properties of hypothetical criterion for degree of belief

- correct belief
- wrongful belief
- correct doubt
- wrongful doubt

Bayesian model selection

$$p(\mathcal{M}|d) = \frac{p(d|\mathcal{M})p(\mathcal{M})}{p(d)} ,$$

- evidence $p(d|\mathcal{M}) = \int d\theta \, p(d|\theta, \mathcal{M})p(\theta)$
- penalty for prior volume $\int d\theta \, p(\theta)$
- normalization constant $p(d) = \sum_i p(d|\mathcal{M}_i)p(\mathcal{M}_i)$ (hard to compute, normally ignored)
- Bayes factor $B_{01} = \frac{p(d|\mathcal{M}_0)}{p(d|\mathcal{M}_1)}$

Doubt

$$\mathcal{D} \equiv p(\mathcal{X}|d) = \frac{p(d|\mathcal{X})p(\mathcal{X})}{p(d)}, \mathcal{R} = \frac{\mathcal{D}}{p(\mathcal{X})}$$

- unknown model $\mathcal{X}$
- estimate evidence $p(d|\mathcal{X})$
- normalization constant computable
  $p(d) = p(d|\mathcal{M})p(\mathcal{M}) + p(d|\mathcal{X})p(\mathcal{X})$
- $\ln \mathcal{R} > 0$: model should be doubted

Estimating Evidence - Bayesian Information Criterion

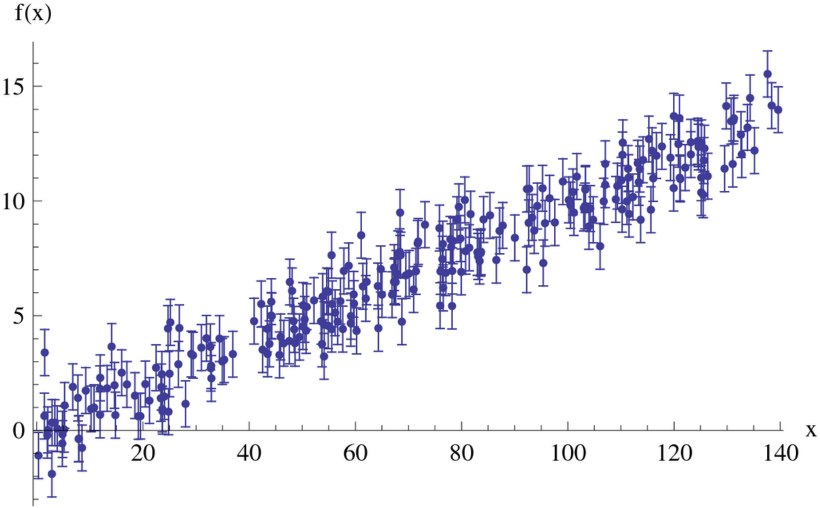
$$p(d|\mathcal{X}) = e^{-\frac{1}{2}\mathrm{BIC}} = \mathcal{L}_{\max} n^{-\frac{1}{2}k}$$

- number of data points n
- number of parameters of the model k
- $\mathcal{L}_{\max} = e^{-\frac{n-k}{2}\frac{\chi^2}{\mathrm{dof}}}$

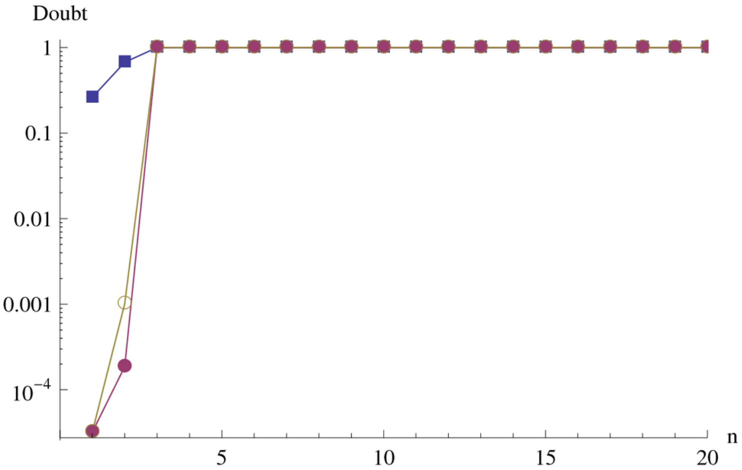
$$\Rightarrow p(d) = p(d|\mathcal{M})p(\mathcal{M}) + p(d|\mathcal{X})p(\mathcal{X})$$

Example

$$f(x)=\frac{1}{10}x$$



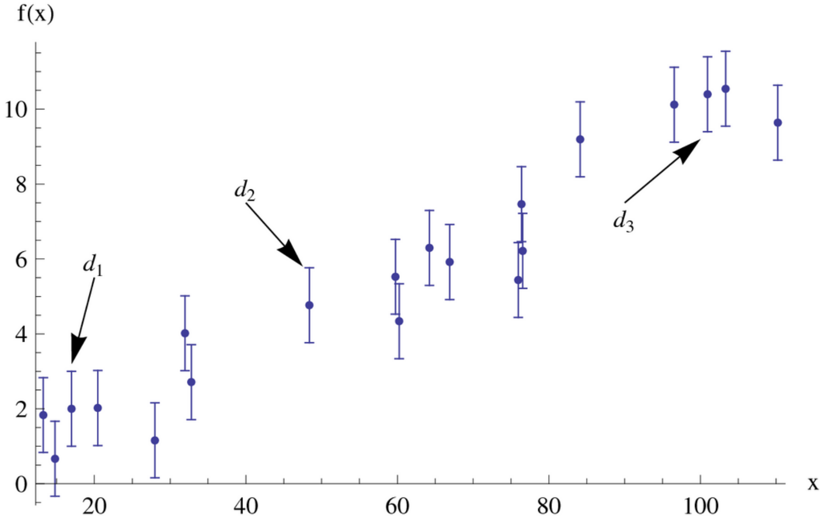
Using the wrong model



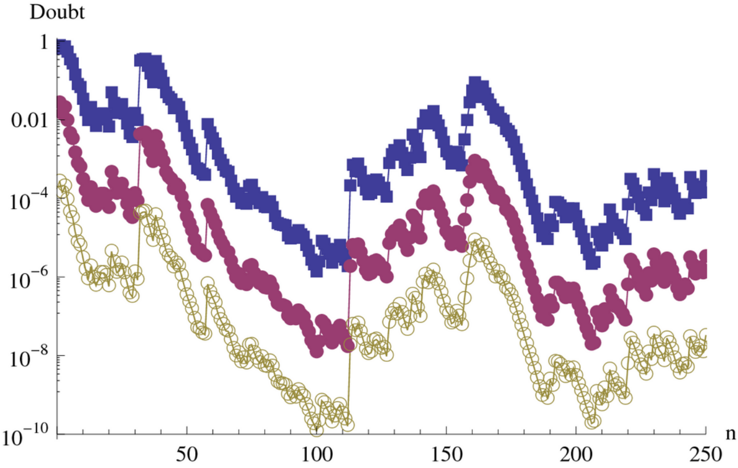
- $p(\mathcal{X}) = 10^{-1}$ (solid blue box)
- $p(\mathcal{X}) = 10^{-5}$ (solid purple circle)

Using the wrong model

$$f(x) = \frac{1}{10}x$$

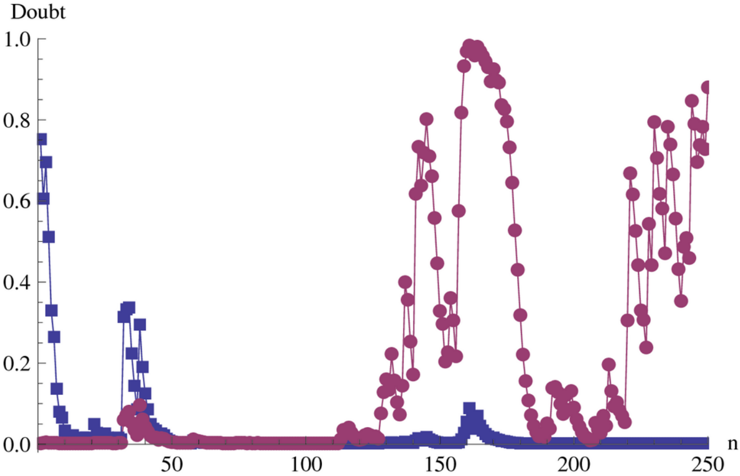


Using the correct model



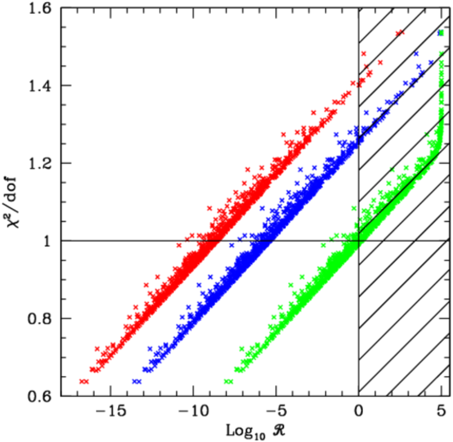
- $p(\mathcal{X}) = 10^{-1}$ (solid blue box)
- $p(\mathcal{X}) = 10^{-3}$ (solid purple circle)
- $p(\mathcal{X}) = 10^{-5}$ (open olive circle)

Estimating $\mathcal{L}_{\text{max}}$



- $\chi^2/\text{dof}=1$: purple circle, too restrictive
- want $\widehat{\mathcal{L}}_{\text{max}} < \mathcal{L}^{\text{true}}_{\text{max}}$ in e.g. 95% of cases
- correct estimator $\widehat{\mathcal{L}}_{\text{max}}$: blue boxes

Performance over 1000 realizations of the data



- green: 50% wrongful doubt
- blue: 5% wrongful doubt
- red: 1% wrongful doubt

Is dark energy the cosmological constant?

- computational cost of posterior $p(d|M) = \int d\theta\, p(d|\theta, M)p(\theta)$
- proper calibration
- compute the (increase in) doubt for $w = -1$ and find

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$$\mathcal{D} = ?$$

Conclusions

- how to compute the degree of belief in given model
- weaknesses of model comparison
- (increase in) doubt $\mathcal{R}, (\mathcal{D})$: single number to quantify degree of (dis)belief
- works well for linear toy model
- currently applying to SN-data