Scanning the Space of Observables

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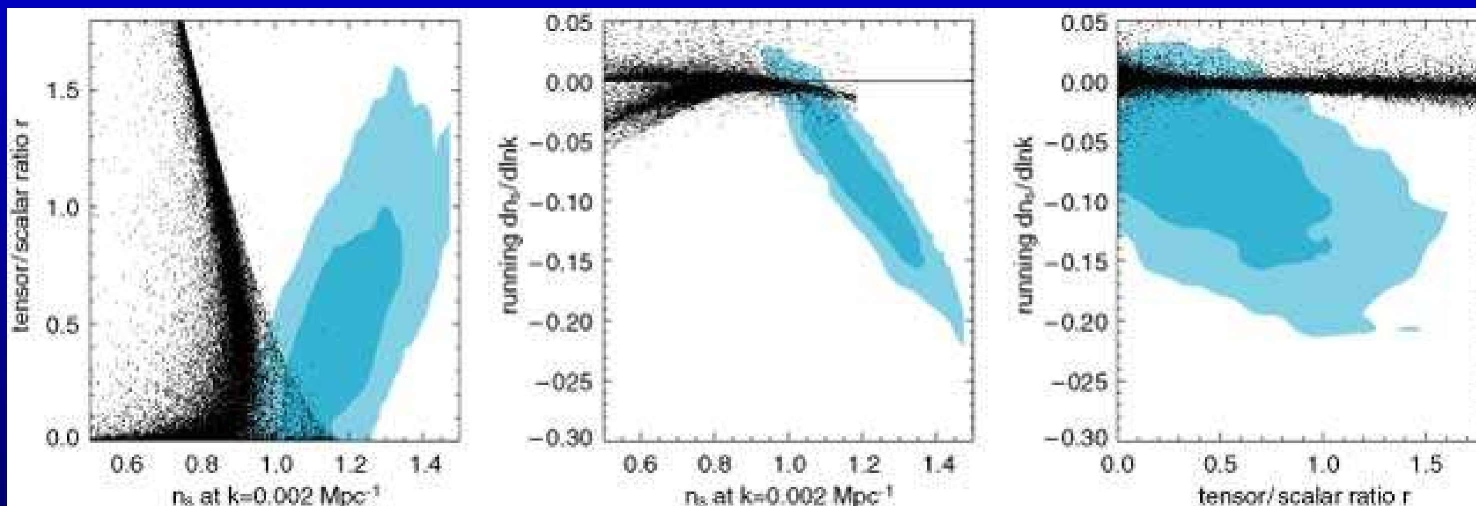
Motivation

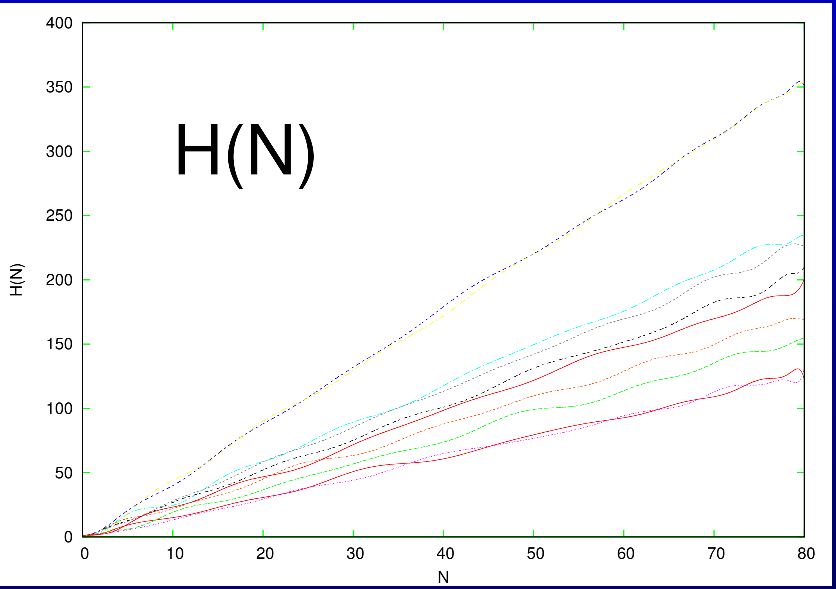


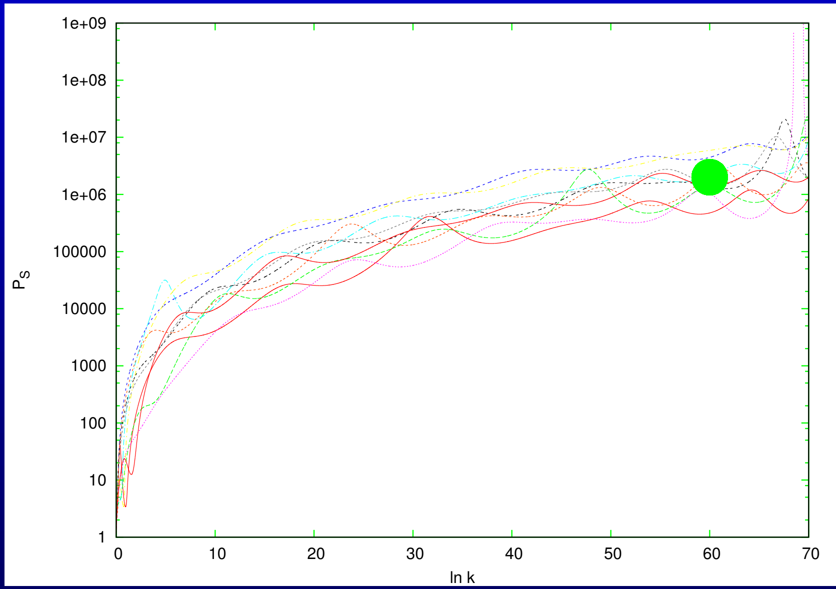
(WMAP, astro-ph/0302225)

- Density of points does not translate into probability
- Different method than RG-equations

reconstruction
$\longleftarrow$

Acceleration **Powerspectrum**





trajectory $\boldsymbol{H(N)}$ $\xrightarrow{\text{map}}$ $$\boldsymbol{P_S = \frac{H^2}{1+q}}$$

(natural object from Hamilton-Jacobi formalism) (mildly broken scale invariance)

Central Object: Trajectory

- N: # of efolds $dN = -H dt$

- Constraints

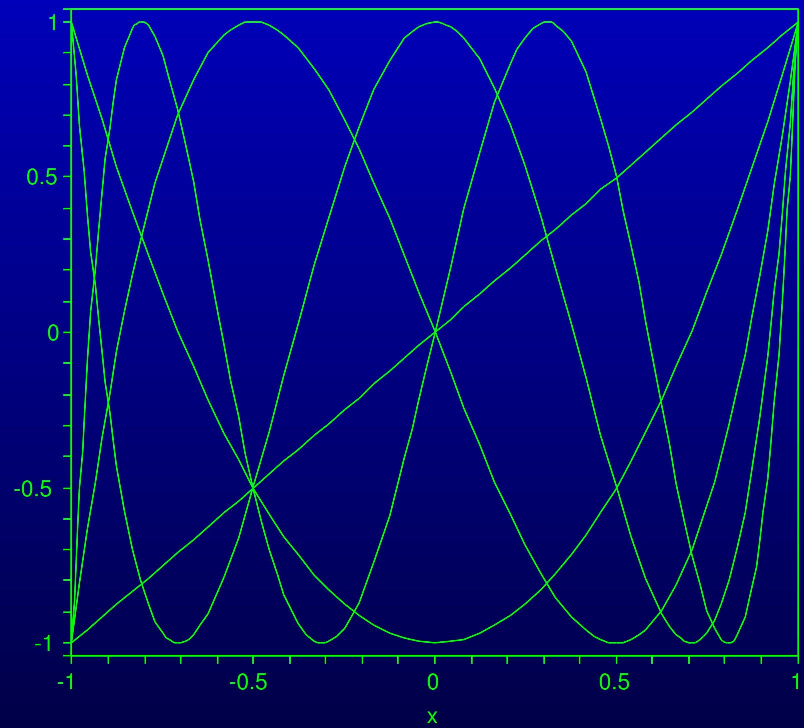
  ⋆ $-1 \leq q \leq 0; \qquad H > 0$

  ⋆ $1 + q = \dfrac{1}{H}\dfrac{dH}{dN}$

- Expansion to arbitrary order

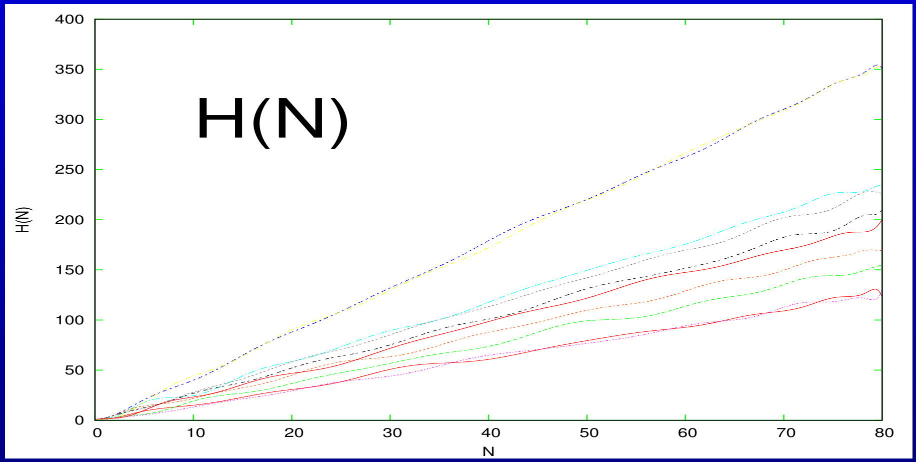
  ⋆ $H(N) = \sum_i c_i T_i(N)$

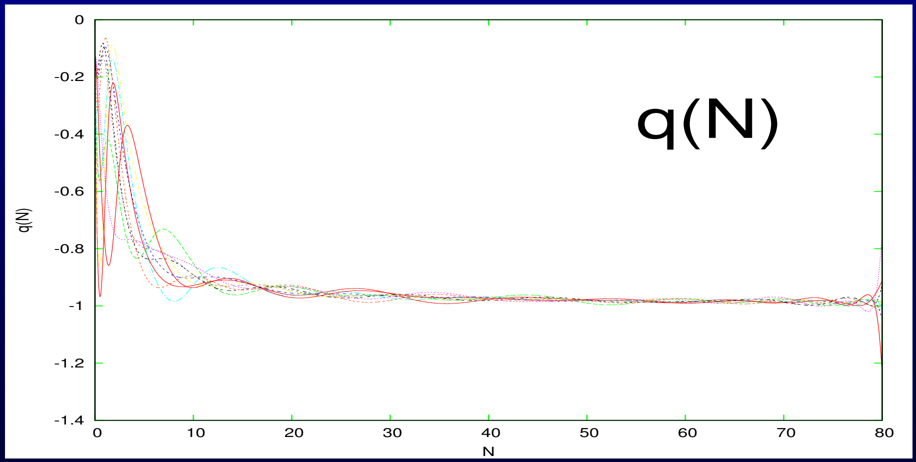
  ⋆ with Chebyshev polynomials $T_i(N)$
  (uniformly best approximation to “true” function)



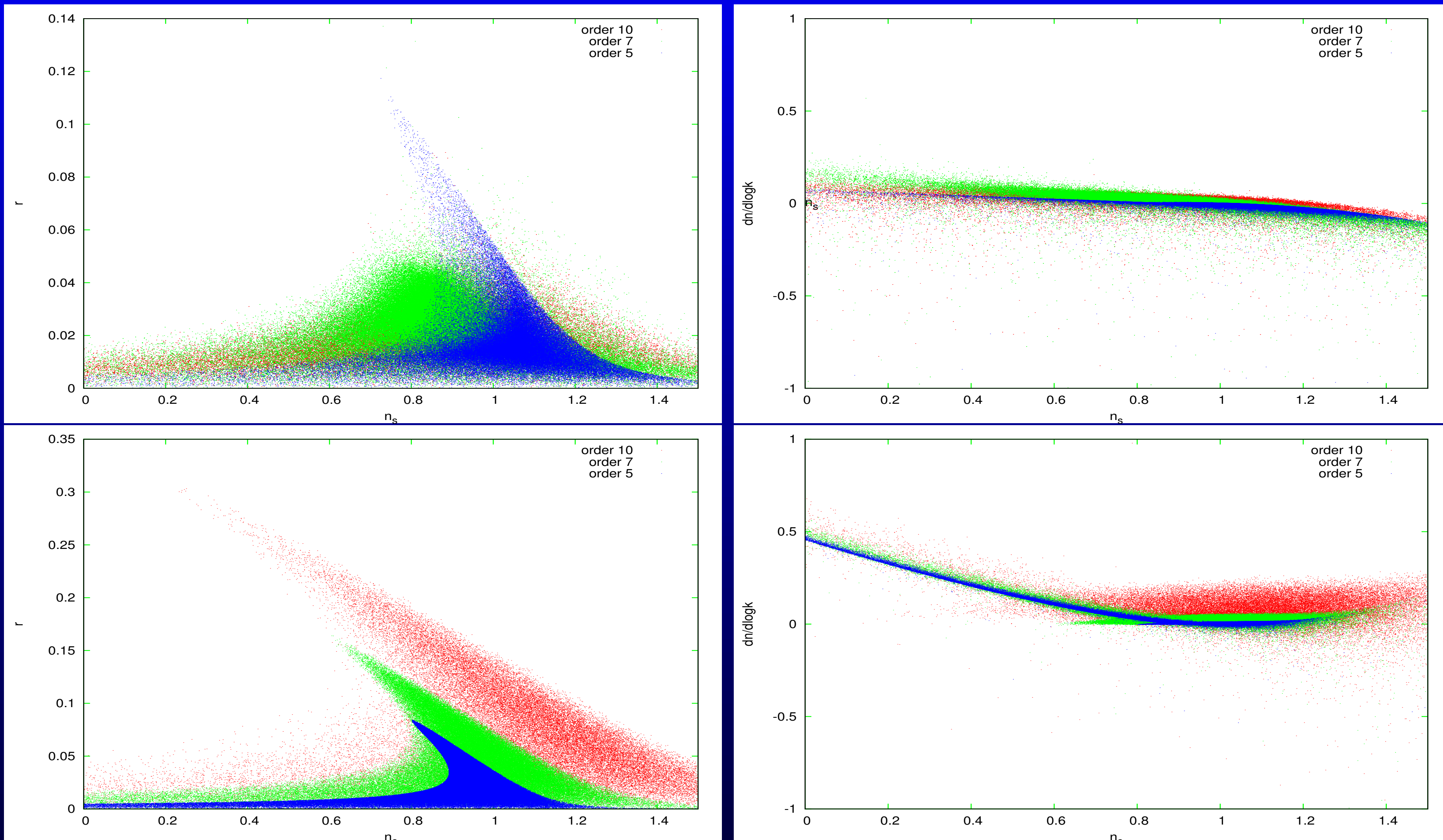
Algorithm

- Initialize $H(N)$ randomly
  - ⋆ by choosing Chebyshev coefficients
  - ⋆ by choosing $H(N)$ at several points N_i
  - ⋆ by choosing $q(N)$ at several points N_i
- Check $-1 \leq q \leq 0$ for $N = 0 \ldots 70$
- Check whether η is small at $N = 70$
- Calculate observables





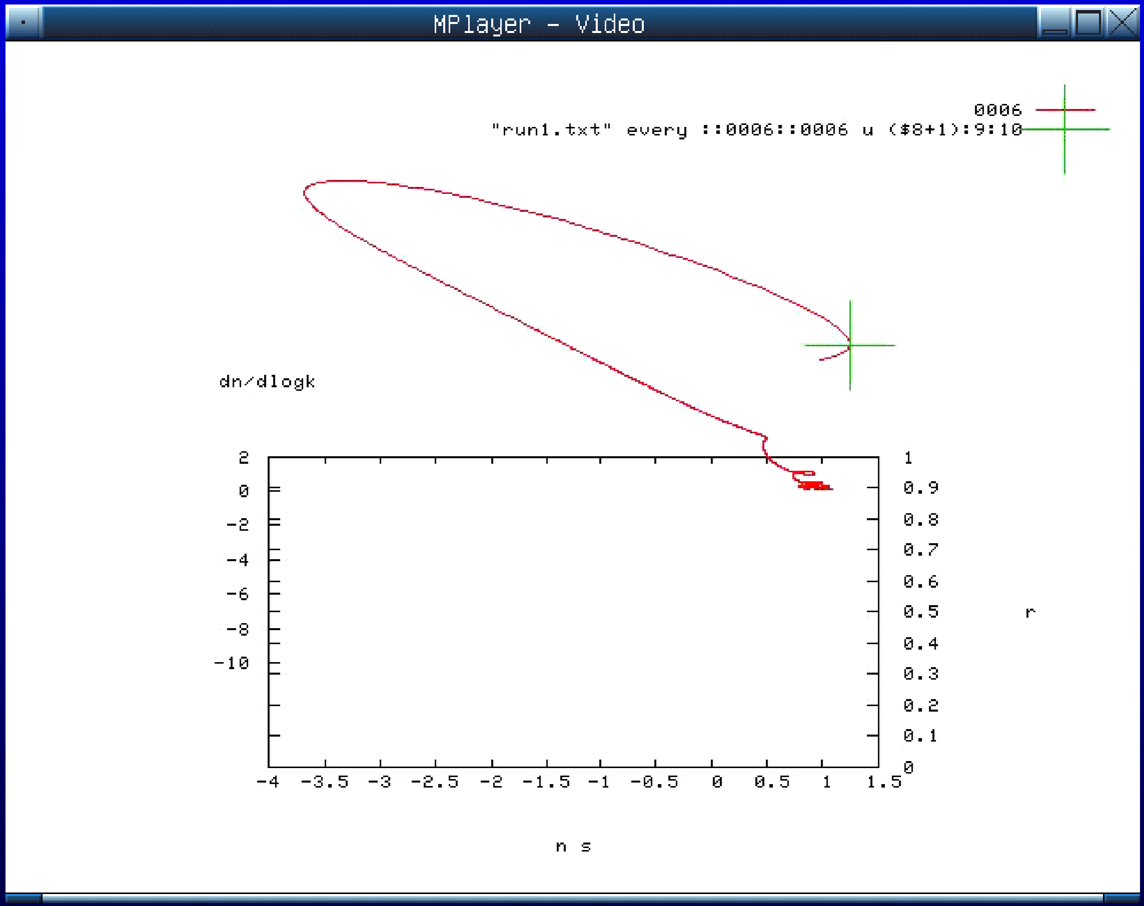
Scanning the Space of Observables



Remarks

- Space of observables opens
- Question of priors
- NOT solving RG-equations
- To lowest order independent of the number of scalar fields
- No scanning over different number of efolds

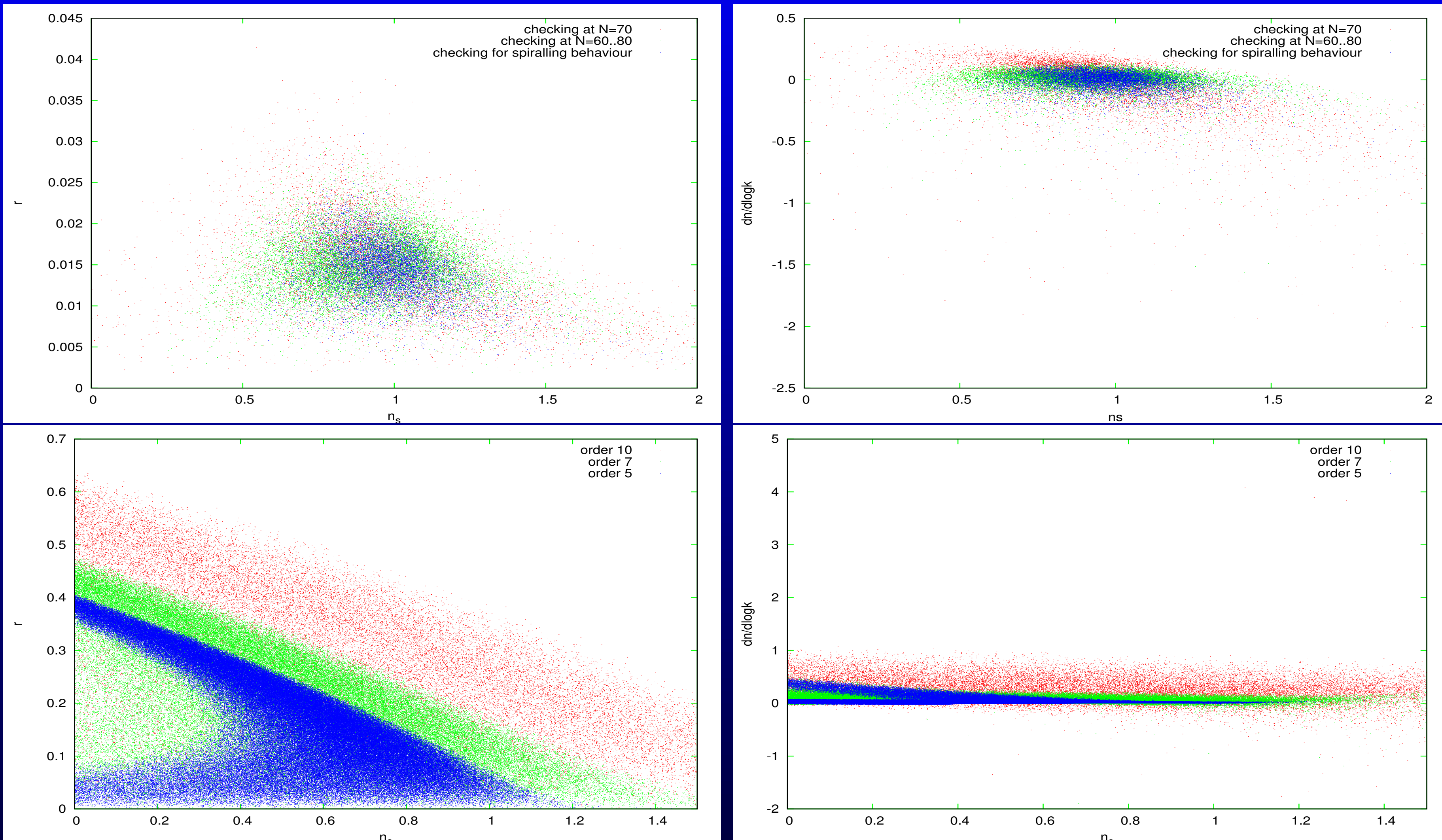
Animation


MPlayer - Video
0006
"run1.txt" every ::0006::0006 u ($8+1):9:10
dn/dlogk
2
0
-2
-4
-6
-8
-10
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
r
-4 -3.5 -3 -2.5 -2 -1.5 -1 -0.5 0 0.5 1 1.5
n s

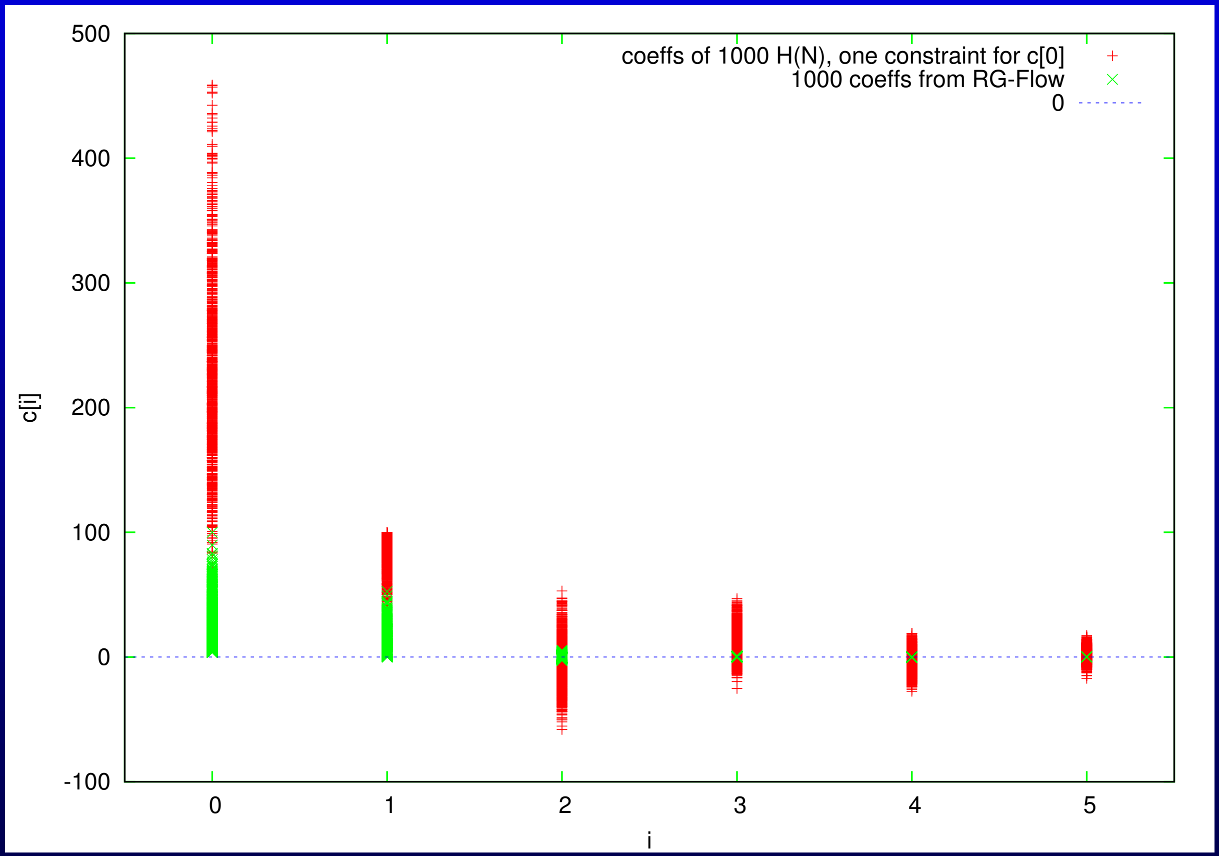
Additional Constraints

mildly broken scale invariance:

- Smallness of observables for $N = 60 \ldots 80$
- Slow-roll parameters stay small for $N = 60 \ldots 80$
- Curves in $(n, r, \frac{dn}{d \log k})$-space bend inwards


checking at N=70
checking at N=60..80
checking for spiralling behaviour
r
n_s
dn/dlogk
ns
order 10
order 7
order 5

Chebyshev transform of $H(N)$



× green dots : traditional RG-flow

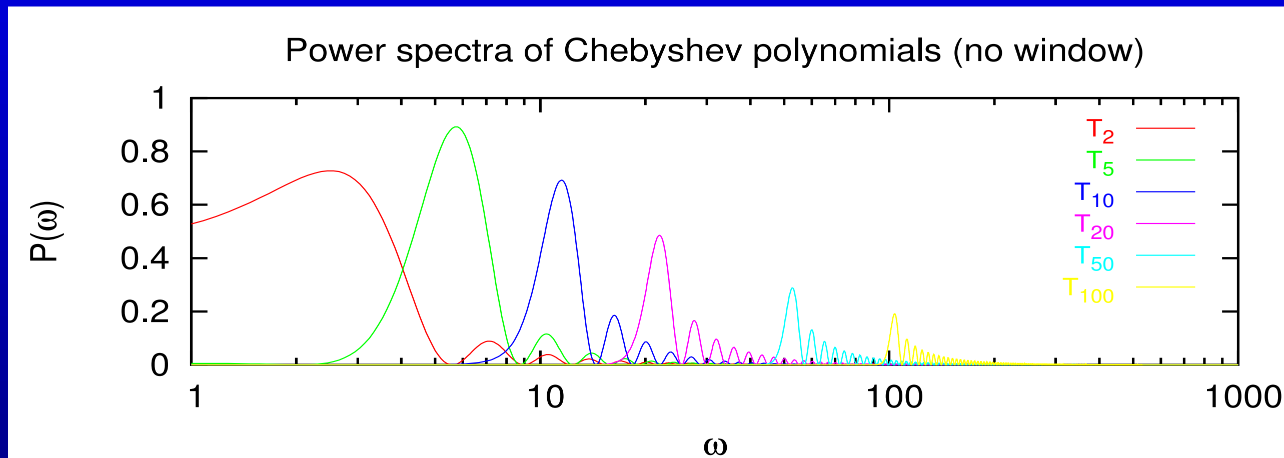
× red dots : our method

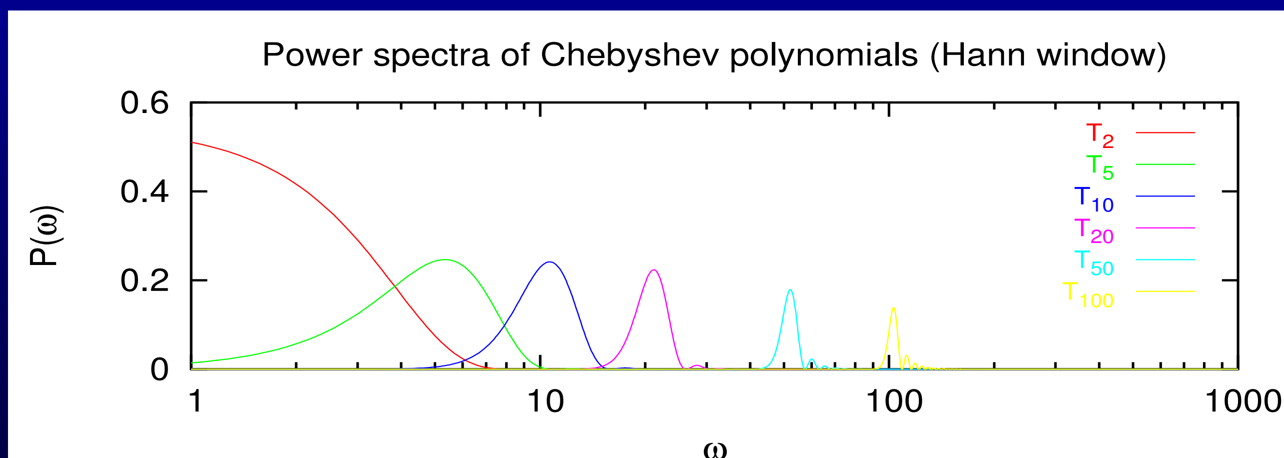
Improvements

$$\ln\left(\frac{k}{0.002Mpc^{-1}}\right) = 62 + \Delta - N + \int_0^N dN' \, \epsilon(N')$$

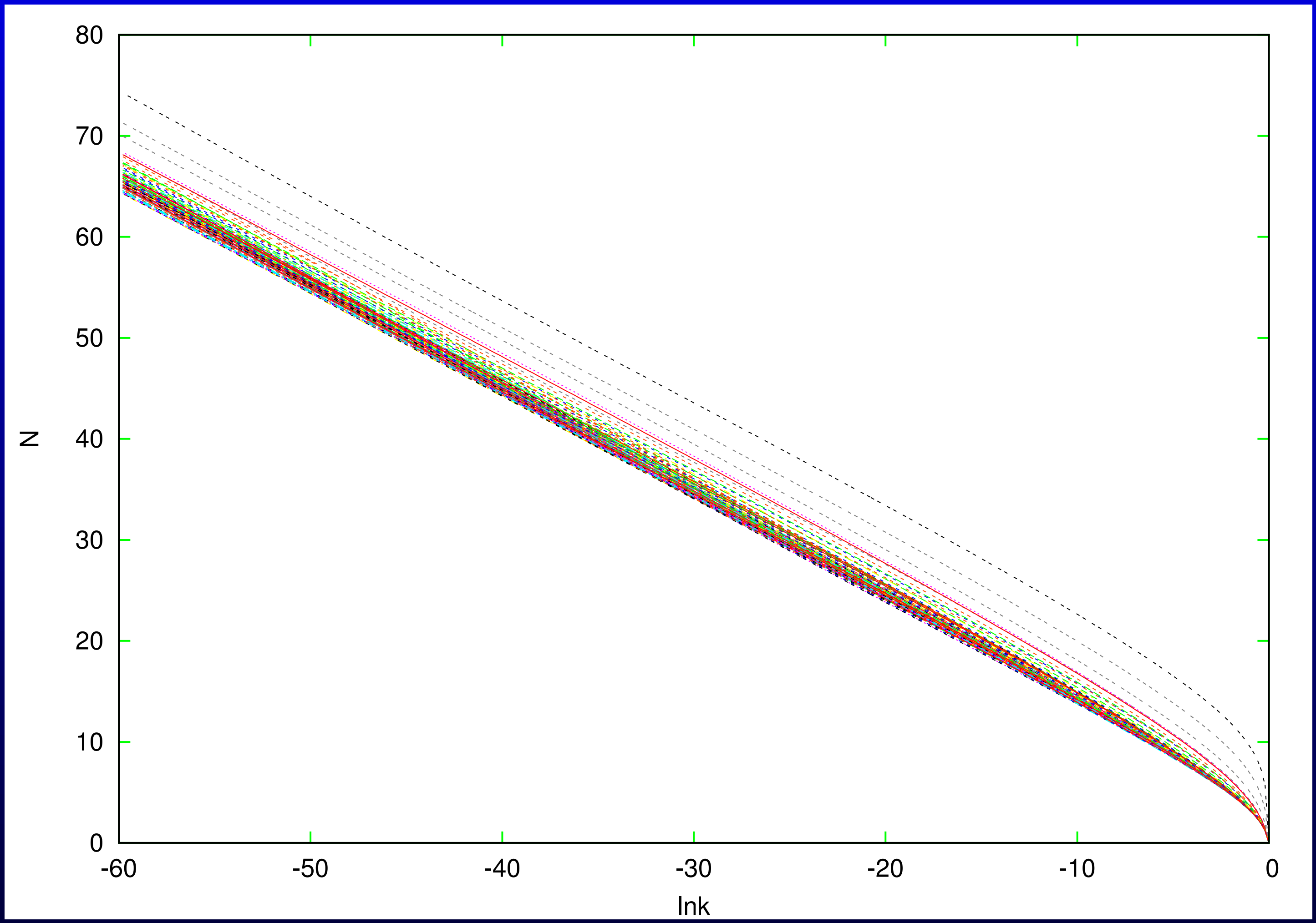
- Δ from reheating/ thermal cosmological history
- Observable horizon today $k = 0.002Mpc^{-1} \Rightarrow N \approx 65$
- What about the $\epsilon(N')$ integral?
- $\Rightarrow$ Better time variable $\ln k$

Effects of Truncation



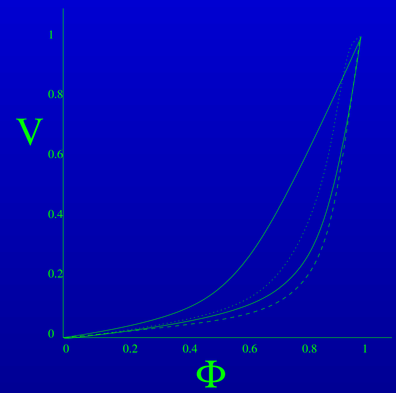


- Truncation = Filtering


80
70
60
50
40
30
20
10
0
N
-60
-50
-40
-30
-20
-10
0
lnk

Reconstructing $V(\phi)$

- Degeneracy if only n_s is known, e.g. (Linde)
  $$\frac{\delta\rho}{\rho} = \frac{2\sqrt{6}}{5\pi}\sqrt{l}\left(\ln\left(\frac{l}{\text{cm}}\right)\right)^{\frac{3}{2}}$$



- Visible in slow-roll

$n_s = 2\eta - 4\epsilon$	1 eq	2 variables
$\frac{dn_s}{d\ln k} = -2({}^2\lambda) + \dots$	2 eq	3 variables
$\vdots$	$\vdots$	$\vdots$
$\frac{d^\ell}{d(\ln k)^\ell} n_s = \dots{}^\ell\lambda\dots$	$(l+1)$ eq	$(l+2)$ variables

- 1 dimensional degeneracy also without slow-roll approximation

Conclusion

- Economic way to scan the space of observables
- To all orders(?) in slow-roll independent of the number of fields
- Increasing the order of the Chebyshev expansions $\longrightarrow$ opening up observables
- Physical priors shrink the space of permissible observables,e.g.
  - ⋆ Renormalizable potentials
  - ⋆ SUGRA
  - ⋆ observational data
  - ⋆ . . .

Slow Roll

SLOW-ROLL

Parameters:

- $1 + q = \epsilon = -\frac{\dot{H}}{H^2}$
- $\eta = \frac{1}{H}\frac{\partial^2}{\partial\phi^2}H$
- ${}^{\ell}\lambda = \frac{1}{H^{\ell}}\left(\frac{\partial H}{\partial \phi}\right)^{\ell-1}\frac{\partial^{(\ell+1)}}{\partial\phi^{(\ell+1)}}H$

Observables

- $n_s = 1 + 2\eta - 4\epsilon$
- $r = \epsilon$
- $\frac{dn}{d\log k} = -2({}^{2}\lambda) + 2\epsilon(5\eta - 4\epsilon)$