Scanning Inflation

J. Richard Bond Carlo R.Contaldi Lev Kofman
Pascal M. Vaudrevange

CITA ICAT Canadian Institute for Theoretical Astrophysics
L'institut canadien d'astrophysique theorique

VELUT ARBOR ÆVO

25.09.2006

Introduction

- Top-down approach to inflation
  seeks to embed it in a fundamental theory
- Bottom-up approach to inflation
  reconstruction of acceleration trajectories

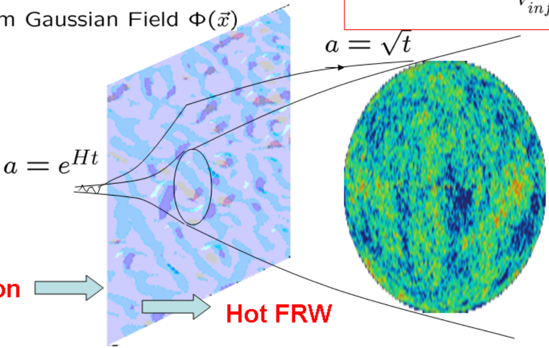
Scalar metric Fluctuations from Inflation

$$ds^2 = -(1-2\Phi)dt^2 + (1+2\Phi)a^2 d\vec{x}^2$$

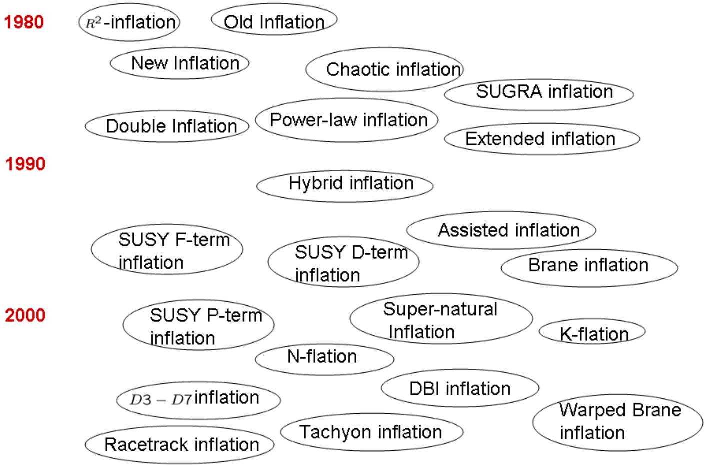
$$\Omega_{tot} = 1$$
$$k^3 \Phi_k^2 \rightarrow P_s = A_s k^{n_s - 1}$$
$$P_T = \frac{H^2}{M_p^2} k^{n_T}$$
$$N = 62 - \ln \frac{10^{16} Gev}{V_{inf}^{1/4}}$$

Initial conditions from Inflation

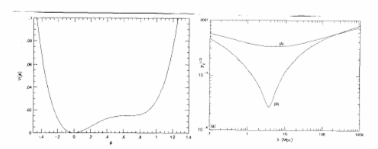
Random Gaussian Field $\Phi(\vec{x})$



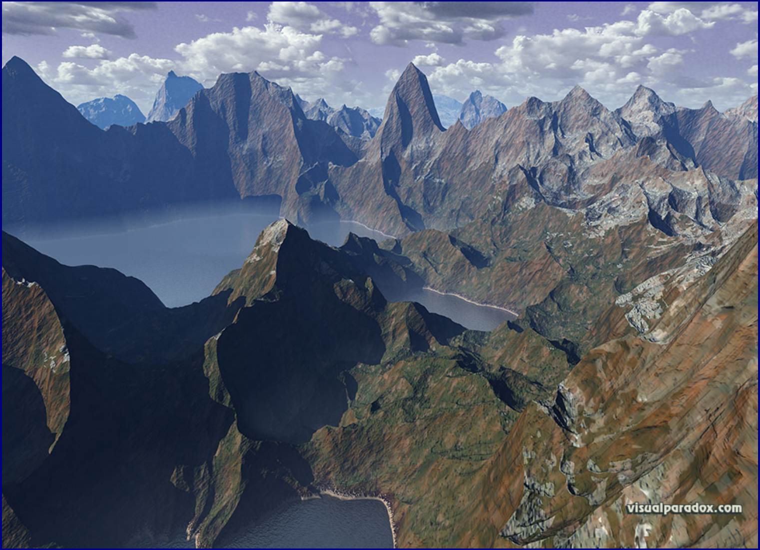
Inflation in the context of ever changing fundamental theory



$$V = m^2\phi^2 + \lambda_1\phi^3 + \lambda_2\phi^4$$



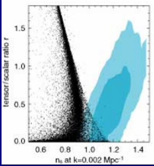
- renormalizable
- different choices of parameters give quite different powerspectra
- but also different shapes of the potential:
  - exponential
  - SUGRA
  - ...


visualparadox.com

Slow-Roll Parameters

- Equations of Motion

$$\begin{aligned} \ddot{\phi} + 3H\dot{\phi} + V' &= 0 \\ \frac{8\pi}{3m_p^2}\left(\frac{1}{2}\dot{\phi}^2 + V\right) &= H^2 \end{aligned}$$

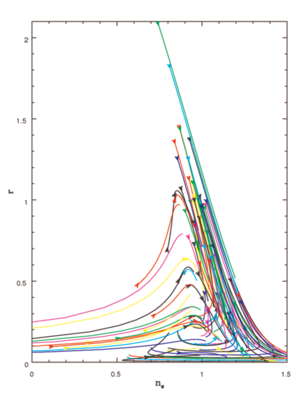


- Slow roll parameters

$$\epsilon = -\frac{\dot{H}}{H^2}, \quad \eta = \frac{m_p^2}{4\pi}\frac{H''}{H}, \quad \zeta^2 = \left(\frac{m_p^2}{4\pi}\right)^2 \frac{H'''H'}{H^2}, \sigma = 2\eta - 4\epsilon$$

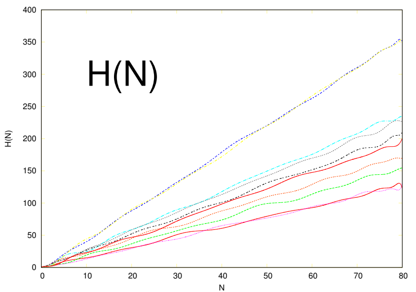
- Flow Equations
- Powerspectra

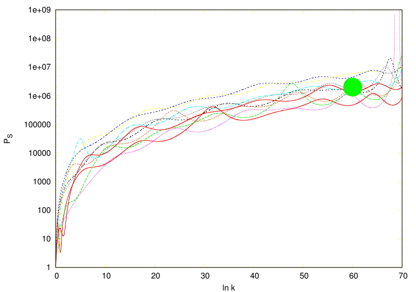
$$P_R \equiv A_S \left(\frac{k}{k_{\rm pivot}}\right)^{n_s - 1} \propto \frac{H^2}{\epsilon}, \quad P_G \equiv A_t \left(\frac{k}{k_{\rm pivot}}\right)^{n_t} \propto H^2$$



varying N moves the points in observable space around

Acceleration $\xleftarrow{\text{reconstruction}}$ **Powerspectrum**



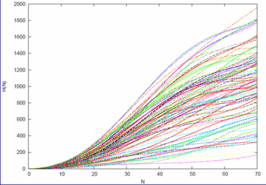


trajectory $H(N)$ $\xrightarrow{\text{map}}$ $P_R \propto \frac{H^2}{\epsilon}$

(natural object from Hamilton-Jacobi formalism)

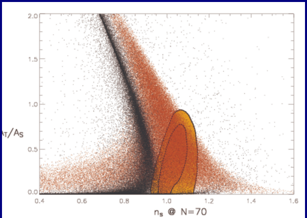
(mildly broken scale invariance)

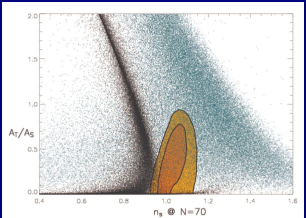
Ensemble of Inflationary Trajectories



various trajectories and time variables

- $\epsilon, H, \ln(P_R), \ln(P_G)$
- $N, \ln(k)$
- $\ldots$

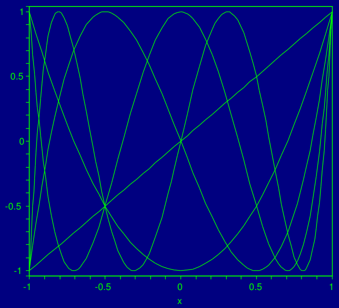




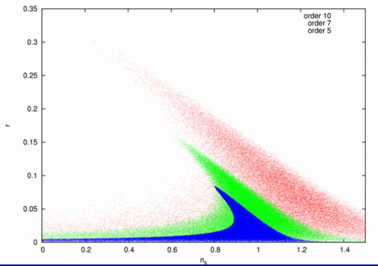
$\Rightarrow$ space opens up, fast

Trajectories

- N: # of efolds $dN = -Hdt$
- Constraints during inflation
  - $0 \leq \epsilon \leq 1$
  - $H > 0$
- at the end of inflation $\epsilon = 1$
- Expansion to arbitrary order
  - $H(N) = \sum_i c_i T_i(x)$
  - with $x = \frac{2N - N_{\max}}{N_{\max}}$, Chebyshev polynomials $T_i(x)$ (uniformly best approximation to "true" function)

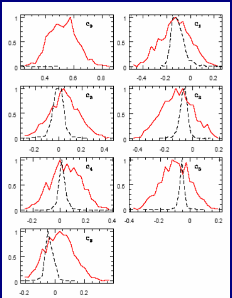


$T_n(\cos(x)) = \cos(nx)$

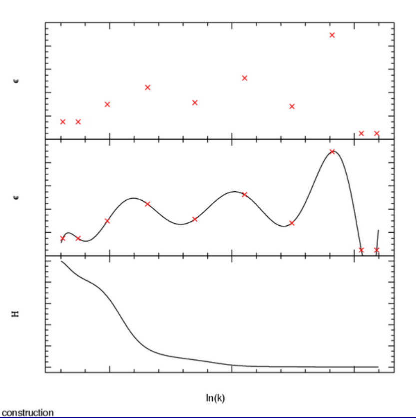


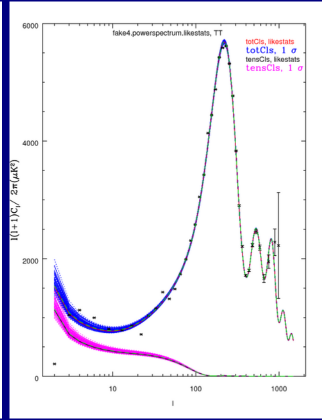
⇐ Space opens more with higher order polynomials

- red c_n
  our method
- black c_n
  Chebyshev-transformed flow equations



Constructing Trajectories





- draw trajectories
- compute likelihood of model given data
- $\Rightarrow$ Markov-Chain Monte-Carlo

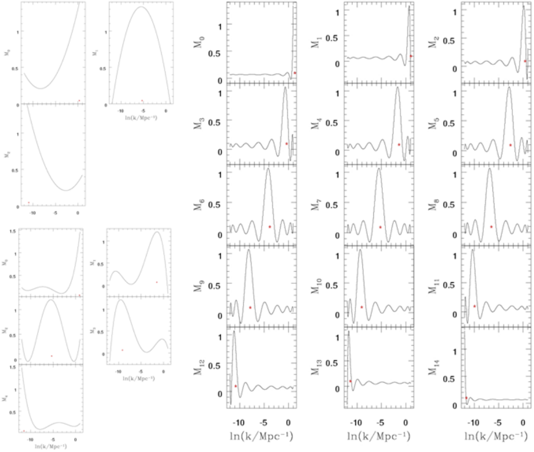
Nodal points

- choose $f(x_i)$ at nodal points x_i
- window function $w_j(x)$:
  $f(x_i) = \sum_j c_j T_j(x_i) \Rightarrow c_j = T_j(x_i)^{-1} f(x_i)$
  $$f(x) = \sum_i c_i T_i(x) = \sum_i f(x_i) \underbrace{\sum_j T_j(x_i)^{-1} T_j(x)}_{w_i}$$
- $f(x) = H, \ln(H), \epsilon, \ln(\epsilon), P_R, \ln(P_R), P_G, \ln(P_G), \ldots$
- $x = N, \ln(k)$

$\Rightarrow \epsilon \in [0, 1]$

Chebyshev nodal modes: order 3, 5, 15 (Fourier at high order)
Chebyshev modes are liner combination

How far does choice of values at nodal points “feel” out?



Displaying Trajectory constraints:

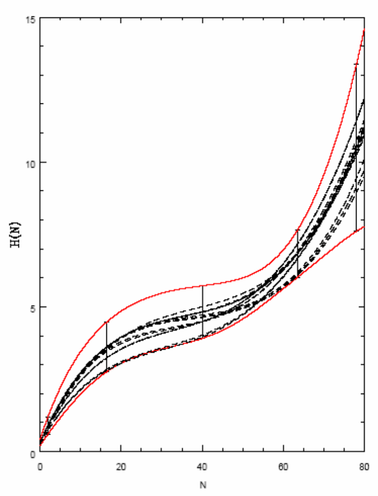
If Gaussian likelihood, compute χ^2 where 68% probability, and follow the ordered trajectories to

$\ln L/L_m = -\chi^2/2$, displaying a uniformly sampled subset.

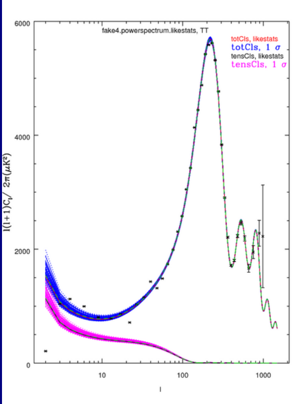
Errors at nodal points in trajectory coefficients can also be displayed.

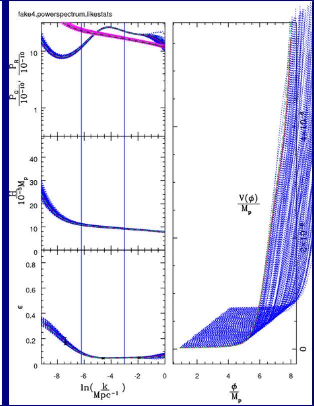
time variable $N \leftrightarrow \ln k$

- N: $N \in [0, 70]$
  but: only interested in observable interval $\Delta N \approx 5$
- $\ln k$: observable $k \in [10^{-4}\mathrm{Mpc}^{-1}, 1\mathrm{Mpc}^{-1}]$



(Re-)Constructing Trajectories





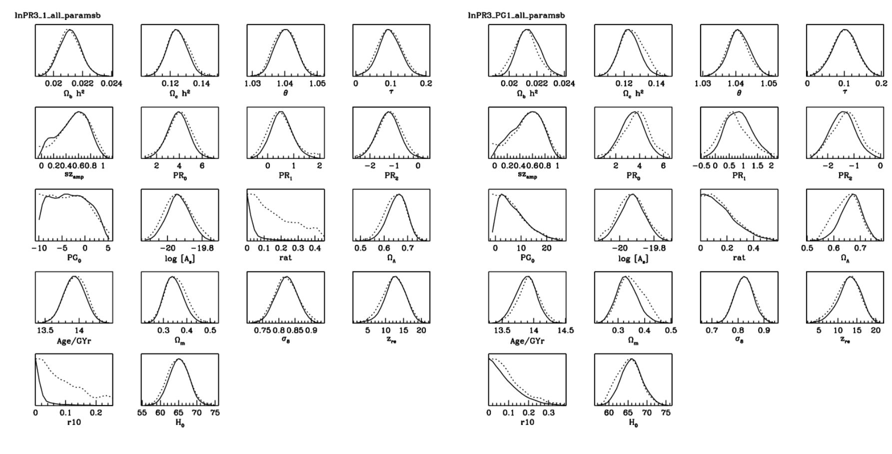
(Re-)Constructing Trajectories

	Mock input value	reconstructed value
c_2	0.0640987	0.074 ± 0.011
c_3	0.0470099	0.049 ± 0.0023
c_4	0.0453124	0.046 ± 0.0016
c_5	0.1725266	0.17 ± 0.011
c_6	0.3512255	0.35 ± 0.015
c_7	0.1578866	0.13 ± 0.039
H_1	144	142 ± 19
Age of universe/GYr	14.1	14.1 ± 0.01
$\Omega_b h^2$	0.01977	0.01976 ± 0.000055
$\Omega_c h^2$	0.12535	0.12522 ± 0.00079
$z_{\rm re}$	15.1	15.1 ± 0.3
σ_8	0.8548	0.8503 ± 0.0044

- Expansion of $\ln(P_R(\ln k)$ to order 3, $\ln(P_G(\ln k))$ to order 1
- Expansion of $\ln(P_R(\ln k)$ to order 3, $P_G(\ln k)$ to order 1
- Expansion of $\epsilon(\ln k)$ to order 10, keeping outmost points fixed

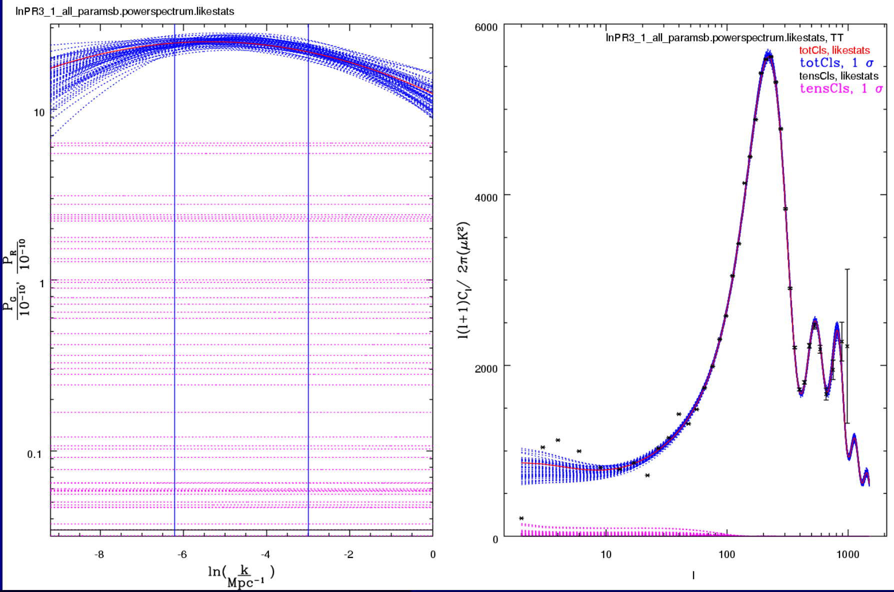
Datasets:

- ACBAR
- BOOMERANG
- CBI
- DASI
- MAXIMA
- VSA
- WMAP
- 2dF
- SDSS


lnPR3_1_all_paramsb
lnPR3_PG1_all_paramsb
$\Omega_b h^2$
$\Omega_c h^2$
θ
τ
sz_{amp}
PR_0
PR_1
PR_2
PG_0
log [A_s]
rat
Ω_Λ
Age/GYr
Ω_m
σ_8
z_{re}
r10
H_0

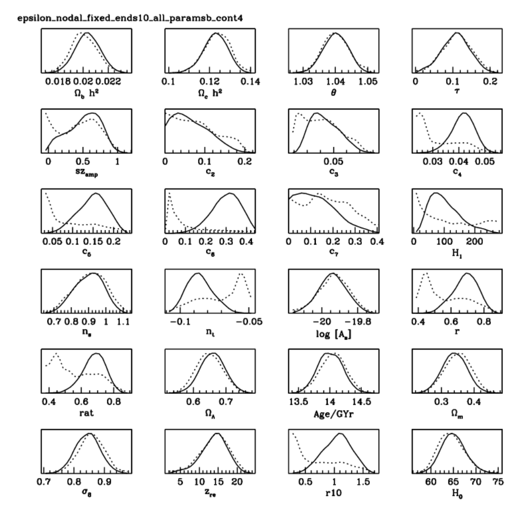
cosmological parameters:

parameter	WMAP3	lnPR 3 lnPG 1	lnPR3 PG1
$\frac{A}{10^{-10}}$	21^{+1}_{-2}	22^{+1}_{-2}	22^{+2}_{-1}
$n_s(0.002Mpc^{-1})$	$1.21^{+0.1}_{-0.15}$	1.16	1.22
$n_s(0.05Mpc^{-1})$	$0.845^{+0.036}_{-0.044}$	0.78	0.79
$\frac{dn_s}{d\ln k}$	$-0.112^{+0.1}_{-0.15}$	-0.117	-0.13
τ	$0.097^{+0.03}_{-0.036}$	$0.096^{+0.03}_{-0.03}$	$0.102^{+0.03}_{-0.03}$
$\dots$	$\dots$	$\dots$	$\dots$

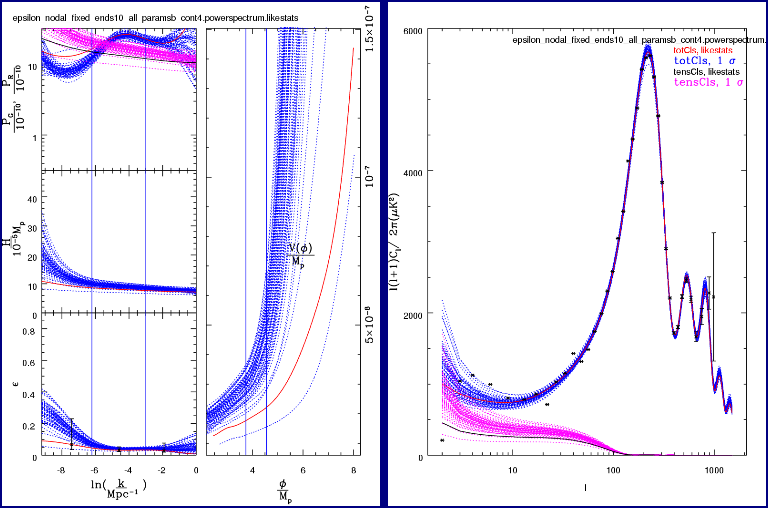

lnPR3_1_all_paramsb.powerspectrum.likestats
10
1
0.1
$\frac{P_G}{10^{-10}}, \frac{P_R}{10^{-10}}$
-8
-6
-4
-2
0
$\ln(\frac{k}{Mpc^{-1}})$
6000
4000
2000
0
lnPR3_1_all_paramsb.powerspectrum.likestats, TT
totCls, likestats
totCls, 1 σ
tensCls, likestats
tensCls, 1 σ
$l(l+1)C_l/\ 2\pi(\mu K^2)$
10
100
1000
l

Expansion of

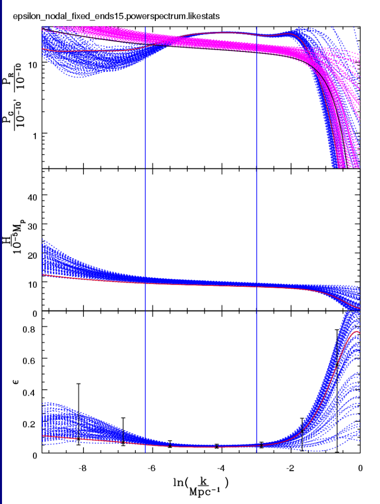
- $\ln(P_R) = \sum_i PR_i T_i(x) \times 10^{-10}$
- $\ln(P_G) = \frac{1}{2} PG_0 T_0(x) \times 10^{-10}$
  $\Rightarrow$ for $PG_0 \in [-10, 5]$, P_G is preferably exponentially small
- $P_G = \frac{1}{2} PG_0$
  $\Rightarrow$ for $PG_0 \in [0, 5]$, P_G is uniform

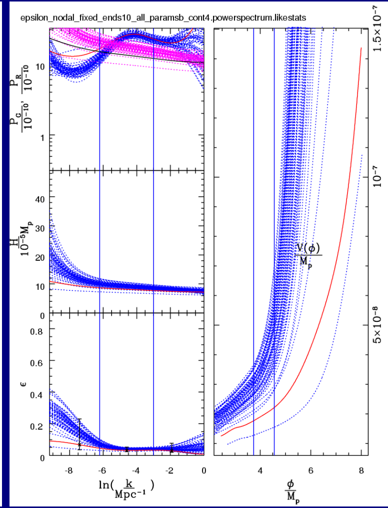


large tensors! (compatible with WMAP3 if running of n_s is allowed)


epsilon_nodal_fixed_ends10_all_paramsb_cont4.powerspectrum.likestats
$\frac{P_G}{10^{-10}}$, $\frac{P_R}{10^{-10}}$
$\frac{H}{10^{-5}M_p}$
ε
$\ln(\frac{k}{Mpc^{-1}})$
$\frac{V(\phi)}{M_p}$
$\frac{\phi}{M_p}$
1.5×10^{-7}
10^{-7}
5×10^{-8}
epsilon_nodal_fixed_ends10_all_paramsb_cont4.powerspectrum.
totCls, likestats
totCls, 1 σ
tensCls, likestats
tensCls, 1 σ
$l(l+1)C_l/2\pi(\mu K^2)$
l

WMAP1 $\leftrightarrow$ WMAP3



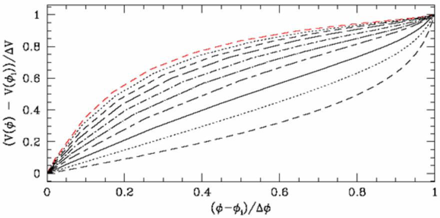


Degeneracy of the Potential Reconstruction

known $P_s(k) \rightarrow$ reconstruct $V(\phi)$

$$P_s(k) = \frac{8\pi H^4}{M_p^4 H'^2}$$

$$V(\phi) = \frac{M_p^4}{32\pi^2}\left(\frac{12\pi}{M_p^2} H^2 - H'^2\right)$$



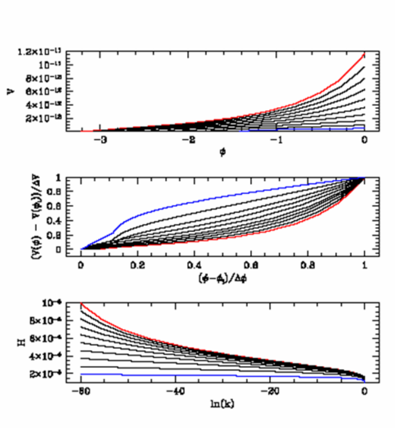
$$\phi - \phi_0 = \frac{M_p^2}{2}\int_{\ln k_0}^{\ln k} d\ln k' \, \frac{\sqrt{P_s(k')}}{H^2(k')}\,\frac{dH}{d\ln k'}$$

$$\frac{dH}{d\log k} = \frac{H^3}{H^2 - \pi M_p^2 P_s(k)}$$

Degeneracy is lifted by fixing $P_{GW} = \frac{H_*^2}{M_p^2}$

Example $P_s(k) = k^{n_s - 1}$

$n_s = 0.98$



Conclusions

- Classical Observables $\leftrightarrow$ power spectra
- Increasing the order of Chebyshevization opens the space of classical observables
- Reconstruction of inflationary trajectories
- Priors
- Degeneracy