Inflationary Trajectories

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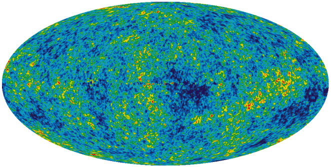
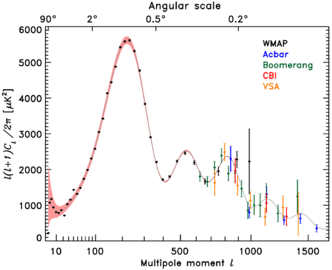
Scanning Inflaton





Goals:

- Reconstruction of Primordial Power Spectra
- Reconstruction of Inflaton Potential

Inflation driven by a scalar field - I

- FLRW metric:

$$ds^2 = dt^2 - a(t)^2(dx^2 + dy^2 + dz^2)$$

- Friedmann Equation

$$3H^2 = \frac{1}{M_p^2}\left(\frac{1}{2}\dot{\phi}^2 + V\right)$$

- Eom for a scalar field

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0$$

- Inflation $\Leftrightarrow 0 \leq \epsilon = -\frac{\dot{H}}{H^2} \leq 1$ ending when $\epsilon = 1$
- Number of efolds N, $dN = -Hdt = \frac{1}{\epsilon - 1} d \ln k$

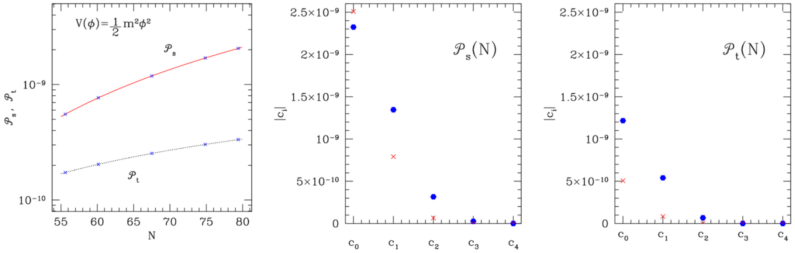
Inflation driven by a scalar field - II

- Power Spectrum of Scalar/ Tensor Perturbations

$$\mathcal{P}_s = \frac{k^3}{2\pi^2}\langle\mathcal{R}_k\mathcal{R}_k\rangle = \frac{1}{8\pi^2}\frac{H^2}{M_p^2\epsilon}\Big|_{k=aH} = A_s\left(\frac{k}{k_{\rm pivot}}\right)^{n_s-1},$$
$$\mathcal{P}_t = \frac{2}{\pi^2}\frac{H^2}{M_p^2}\Big|_{k=aH} = A_t\left(\frac{k}{k_{\rm pivot}}\right)^{n_t}$$

- standard parametrization
  - amplitude A_s
  - scalar/ tensor spectral index n_s, n_t
  - running of the spectral index $n_{\rm run} = dn_s/d\ln k$
  - tensor scalar ratio $r = \mathcal{P}_t/\mathcal{P}_s$

Chebyshev expansion

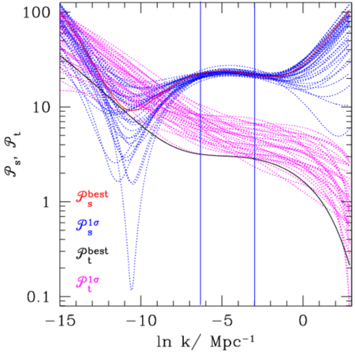


trajectory expansion of
$f \in \{\mathcal{P}_s, \mathcal{P}_t, \ln \mathcal{P}_s, \ln \mathcal{P}_t, \epsilon, n_t, \ln(\epsilon), \ln(-n_t), \dots\}$

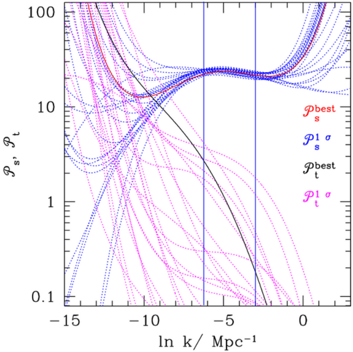
- $f(x) = \sum c_j T_j(x) = \sum f(x_j) w_j(x)$
- $T_j(\cos(x)) = \cos(jx)$
- power spectra: solid line: $\mathcal{P}_s$, dotted line: $\mathcal{P}_t$
- coefficients: $V = \lambda\phi^4$, $V = \frac{1}{2}m^2\phi^2$

MCMC Reconstruction of Primordial Power Spectra

CMB+ LSS data sets



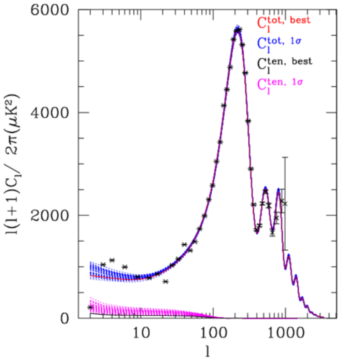
$\mathcal{P}_s, \mathcal{P}_t$ to order 5
$r = 0.228^{+0.037}_{-0.055}$



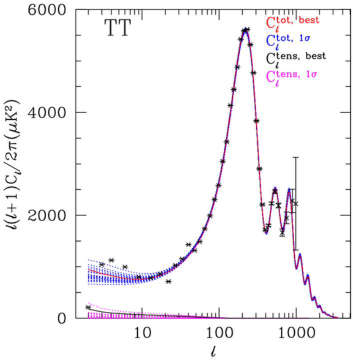
$\ln \mathcal{P}_s, \ln \mathcal{P}_t$ to order 5
$r < 0.075$

using standard parametrization: $r < 0.36$

MCMC Reconstruction of Primordial Power Spectra



$\mathcal{P}_s, \mathcal{P}_t$ to order 5

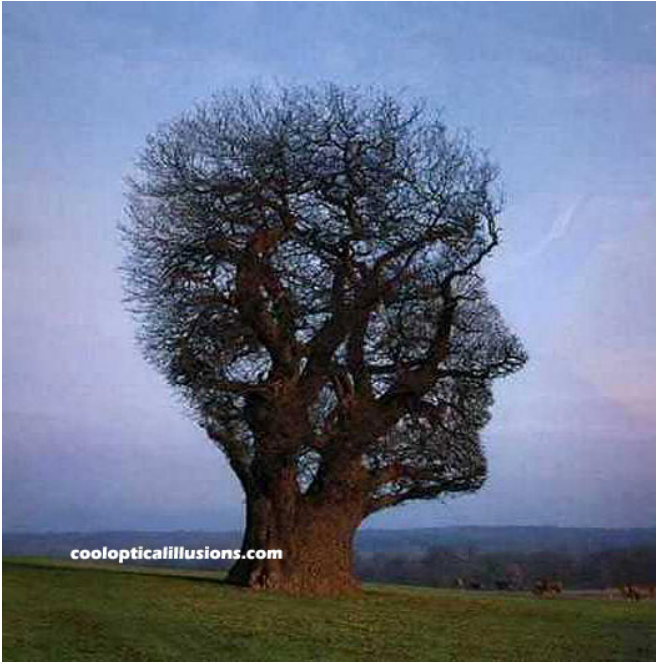


$\ln \mathcal{P}_s, \ln \mathcal{P}_t$ to order 5

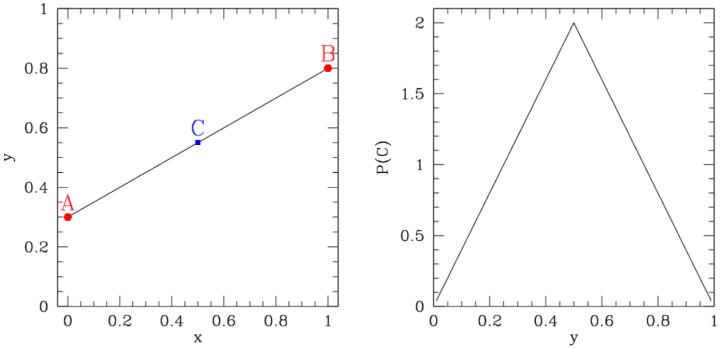
Reconstructing Simulated Data Sets

Parameter	Initial	uniform prior on $\ln \mathcal{P}_s, \ln \mathcal{P}_t$		uniform prior on $\mathcal{P}_s, \mathcal{P}_t$	
		Planck	CMBPol	Planck	CMBPol
n_s	0.96	$0.9595^{+0.0035}_{-0.0035}$	$0.9591^{+0.0024}_{-0.0024}$	$0.9548^{+0.0051}_{-0.0052}$	$0.957^{+0.0023}_{-0.0023}$
$n_{\rm run}$	-0.0551	$-0.055^{+0.0066}_{-0.0065}$	$-0.056^{+0.0034}_{-0.0034}$	$-0.0511^{+0.0094}_{-0.0095}$	$-0.0578^{+0.0028}_{-0.0029}$
$\ln[A_s]$	-19.9578	$-19.958^{+0.0016}_{-0.0017}$	$-19.9577^{+0.0011}_{-0.0011}$	$-19.9582^{+0.0028}_{-0.0026}$	$-19.9572^{+0.0011}_{-0.001}$
r	0.1	$< 0.077 (95\% CL)$	$0.0852^{+0.0081}_{-0.008}$	$< 0.13 (95\% CL)$	$0.0925^{+0.0047}_{-0.0047}$
n_s	0.96	$0.9595^{+0.0036}_{-0.0036}$	$0.959^{+0.0024}_{-0.0023}$	$0.9549^{+0.0043}_{-0.0044}$	$0.957^{+0.0022}_{-0.0022}$
$n_{\rm run}$	-0.0551	$-0.0558^{+0.0065}_{-0.0067}$	$-0.0561^{+0.0034}_{-0.0033}$	$-0.0517^{+0.0084}_{-0.0086}$	$-0.0578^{+0.0028}_{-0.0028}$
$\ln[A_s]$	-19.9578	$-19.958^{+0.0017}_{-0.0017}$	$-19.9577^{+0.0012}_{-0.0011}$	$-19.9583^{+0.0023}_{-0.0023}$	$-19.9572^{+0.0011}_{-0.001}$
r	0.01	$< 0.0088 (95\% CL)$	$0.0072^{+0.0014}_{-0.0015}$	$< 0.042 (95\% CL)$	$0.00868^{+0.00092}_{-0.00092}$

- Simulated data sets with Planck's and CMBPol's errors
- keeping other parameters fixed

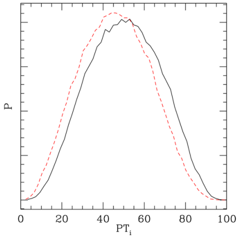

coolopticalillusions.com

Influence of Priors

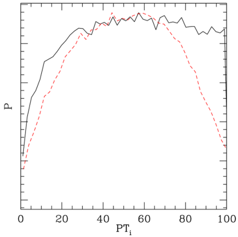


- uniform prior at A, B
- non-uniform at C!

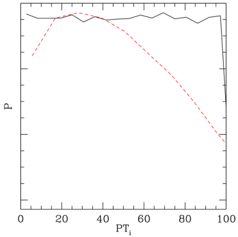
$\mathcal{P}_s, \mathcal{P}_t$ order 5



monotonic $\mathcal{P}_t$



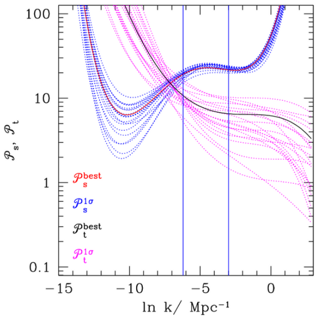
non-monotonic $\mathcal{P}_t$



non-monotonic ln $\mathcal{P}_t$

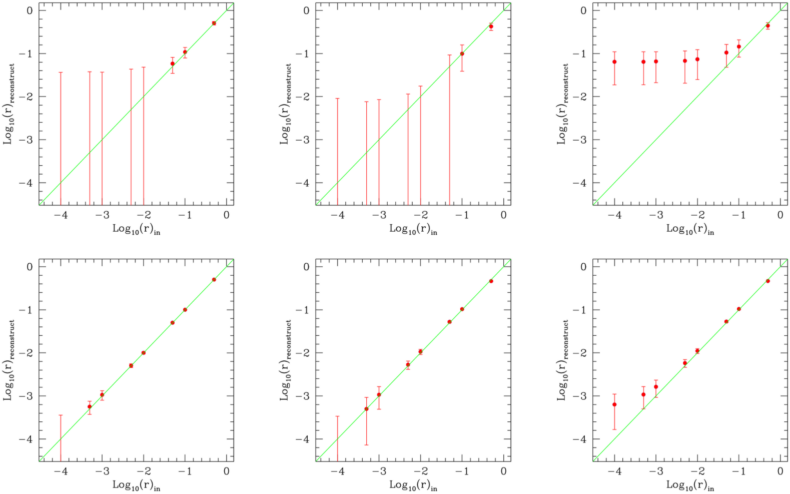
- uniform prior on $\mathcal{P}_T$
- dashed red : in between nodal points
- solid black: at nodal points

$\ln \mathcal{P}_s, \ln \mathcal{P}_t$ order 5, uniform in $\mathcal{P}_s, \mathcal{P}_t$



Parameter	Value
$\Omega_b h^2$	$0.02202^{+0.00079}_{-0.0008}$
$\Omega_c h^2$	$0.1177^{+0.0058}_{-0.0061}$
θ	$1.0422^{+0.0031}_{-0.0031}$
τ	$0.107^{+0.034}_{-0.033}$
n_s	$0.936^{+0.036}_{-0.035}$
$n_{\rm run}$	$0.034^{+0.03}_{-0.029}$
$\ln[A_s]$	$-19.925^{+0.072}_{-0.072}$
r	$< 0.41 (95\% CL)$

ϵ trajectories

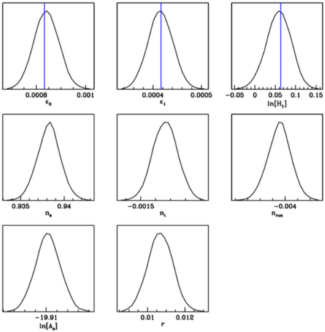


standard

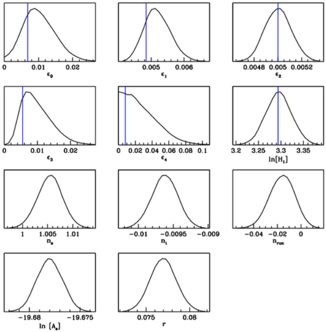
ln ϵ5

ln ϵ5 uniform ϵ

Reconstruction of Simulated Data Sets



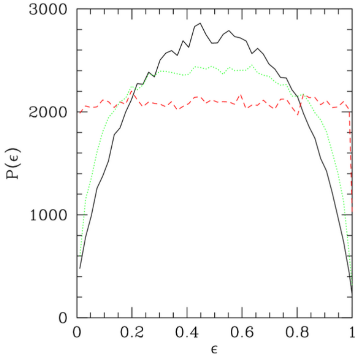
ϵ order 2



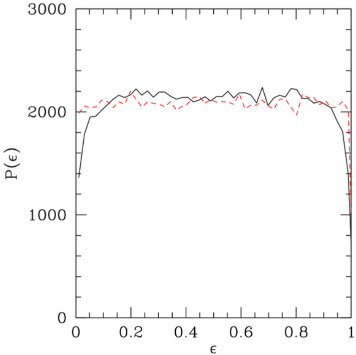
ϵ order 5

keeping other parameters fixed

Acceleration trajectories order 5



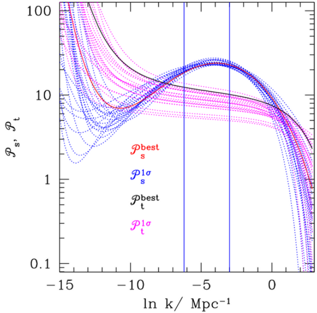
ϵ



$\ln(-n_t)$

- dashed red line : at nodal points
- dotted green line : at nodal points with rejection
- solid black line : in between nodal points

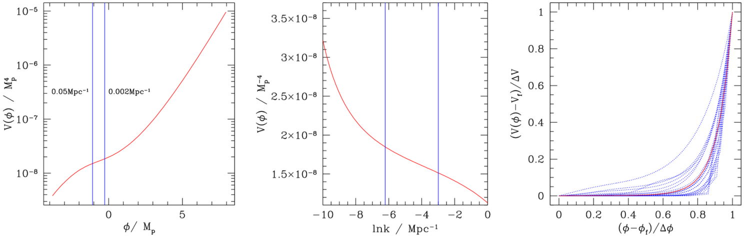
$\ln(-n_t)$ to order 5, uniform prior on ϵ



Parameter	Value
$\Omega_b h^2$	$0.02234^{+0.00085}_{-0.00081}$
$\Omega_c h^2$	$0.1263^{+0.0068}_{-0.0067}$
θ	$1.045^{+0.0033}_{-0.0033}$
τ	$0.1^{+0.03}_{-0.03}$
H_1	$1.82^{+0.14}_{-0.14}$
n_s	$0.885^{+0.04}_{-0.04}$
n_t	$-0.049^{+0.013}_{-0.012}$
n_{run}	$-0.121^{+0.032}_{-0.031}$
$log[A_s]$	$-19.899^{+0.069}_{-0.071}$
r	$0.39^{+0.099}_{-0.1}$

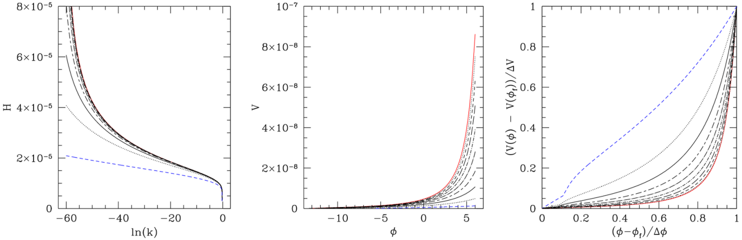
MCMC reconstruction of the inflaton potential

$\ln(-n_t)$ to order 5 with uniform prior on ϵ



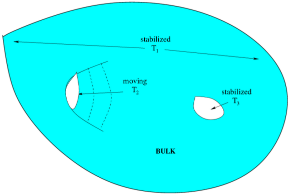
$$
\begin{aligned}
\frac{d\ln H}{d\ln k} &= -\frac{\epsilon}{1-\epsilon}, \quad \frac{d\phi}{d\ln k} = -\frac{\sqrt{2\epsilon}}{1-\epsilon} \\
V(\ln k) &= 3H(\ln k)^2\left(1-\frac{1}{3}\epsilon(\ln k)\right)
\end{aligned}
$$

Degeneracy of the potential reconstruction



- $$\frac{dH}{d\ln k} = \frac{H^3(k)}{H^2(k) - 8\pi^2 M_p^2 \mathcal{P}_s(k)}$$
- H_1 from $\mathcal{P}_t = \dfrac{2}{\pi^2}\dfrac{H^2}{M_p^2}$

Roulette Inflation



$$\frac{\mathcal{K}}{M_P^2} = -2\ln\left(\mathcal{V}_s + \frac{\xi g_s^{\frac{3}{2}}}{2e^{\frac{3\phi}{2}}}\right) + \ldots, \mathcal{V}_s = \frac{1}{9\sqrt{2}}\left(\tau_1^{3/2} - \tau_2^{3/2}\right).$$

$$\hat{W} = \frac{g_s^{\frac{3}{2}} M_P^3}{\sqrt{4\pi}}\left(W_0 + \sum A_i e^{-a_i T_i}\right)$$

$$V(T,\bar{T}) = e^{\mathcal{K}/M_P^2}\left(\mathcal{K}^{i\bar{j}} D_i \hat{W} D_{\bar{j}} \bar{\hat{W}} - \frac{3}{M_P^2}\hat{W}\bar{\hat{W}}\right) + \text{D-terms}.$$

- SUGRA approximation to large volume IIB-compactification by Conlon, Quevedo, 2005
- Kähler moduli stabilized by non-perturbative effects

⇒ Resulting potential

$$V(\mathcal{V}, T_2, ..., T_n) =$$

$$\frac{12W_0^2\xi}{(4\mathcal{V}-\xi)(2\mathcal{V}+\xi)^2} + \sum_{i=2}^{n} \frac{12e^{-2a_i\tau_i}\xi A_i^2}{(4\mathcal{V}-\xi)(2\mathcal{V}+\xi)^2} + \frac{16(a_iA_i)^2\sqrt{\tau_i}e^{-2a_i\tau_i}}{3\alpha\lambda_2(2\mathcal{V}+\xi)}$$

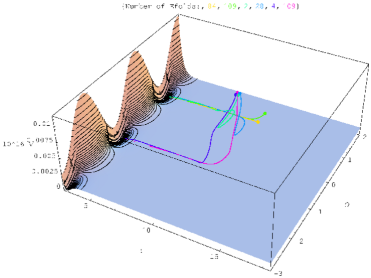
$$+ \frac{32e^{-2a_i\tau_i}a_iA_i^2\tau_i(1+a_i\tau_i)}{(4\mathcal{V}-\xi)(2\mathcal{V}+\xi)} + \frac{8W_0A_ie^{-a_i\tau_i}\cos(a_i\theta_i)}{(4\mathcal{V}-\xi)(2\mathcal{V}+\xi)}\left(\frac{3\xi}{(2\mathcal{V}+\xi)} + 4a_i\tau_i\right)$$

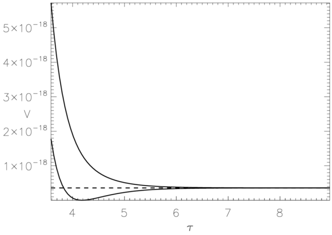
$$+ \sum_{\substack{i,j=2 \\ i<j}}^{n} \frac{A_iA_j\cos(a_i\theta_i - a_j\theta_j)}{(4\mathcal{V}-\xi)(2\mathcal{V}+\xi)^2}\, e^{-(a_i\tau_i+a_j\tau_j)}\left[32(2\mathcal{V}+\xi)(a_i\tau_i + a_j\tau_j\right.$$

$$\left. +2a_ia_j\tau_i\tau_j) + 24\xi\right]$$

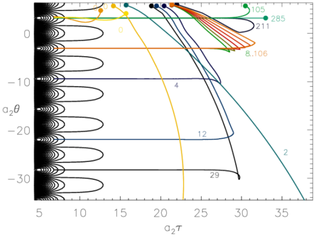
$$T_i = \tau_i + i\theta_i$$

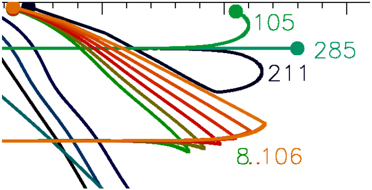
$$\Rightarrow V(\tau,\theta) \approx \frac{8(a_2A_2)^2\sqrt{\tau}e^{-2a_2\tau}}{3\alpha\lambda_2\mathcal{V}_m} - \frac{4W_0a_2A_2\tau e^{-a_2\tau}\cos(a_2\theta)}{\mathcal{V}_m^2} + \Delta V\,,$$





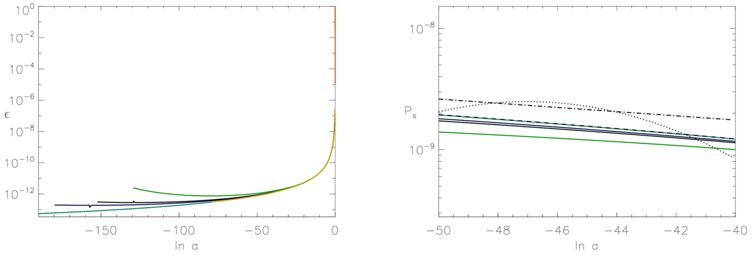
- Roulette Inflation ⇔ Racetrack scenario ⇔ Kähler modulus inflation





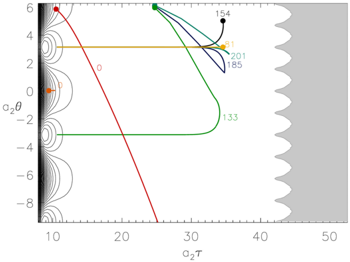
- axionic direction
- initial conditions
- large number of efolds
- bifurcation points
- isocurvature perturbations?
- $N = 40 \ldots 50$

(from $N(k) = 62 - \ln \frac{k}{6.96 \times 10^{-5}\,\mathrm{Mpc}^{-1}} + \Delta$, with $\Delta = -\ln \frac{10^{16}\mathrm{GeV}}{V_k^{1/4}} + \frac{1}{4} \ln \frac{V_k}{V_{\mathrm{end}}} - \frac{1}{3} \ln \frac{V_{\mathrm{end}}^{1/4}}{\rho_{\mathrm{reh}}^{1/4}}$)



- extremely low $\mathcal{P}_t$
- template $\mathcal{P}_s \equiv A_S(k/k_{\rm pivot})^{n_s-1}$ with
  - pivot point $N = 45$
  - dash-dot line: $n_s = 0.95, n_{\rm run} = 0$
  - dotted line: $n_s = 0.95$, $n_{\rm run} = -0.055$

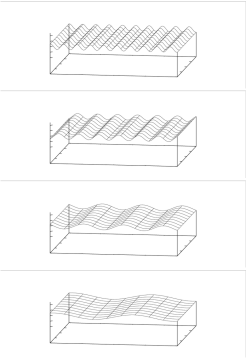
Stochastic regime



- shaded region: quantum kicks $\gg$ classical motion
- self reproduction
- distribution of initial values
  - entering from stochastic region
  - raining down from other holes settling to their minima

Trans-Planckian effects in the Milne Universe

- FRW metric
  $ds^2 = a(\eta)^2 \left(d\eta^2 - dr^2 - f(r)^2 d\Omega^2\right)$
- Physical scales grow like $\frac{k}{a}$
- Probing Trans-Planckian Regime ?
- Consistency?
- Milne Universe $f(r) = \sinh r$, $a(\eta) = e^{\eta}$
- Scalar field $\mathcal{L} = \phi^{*,\mu}\phi_{,\mu} - (m^2 - \frac{R}{6})\phi^*\phi$
- Choice of vacuum
  - Adiabatic Minkowsky Vacuum
  - Conformal Vacuum



- Calculate $\langle 0|T_{\mu\nu}|0\rangle$
  - Minkowsky vacuum: $\langle 0|T_{\mu\nu}|0\rangle = 0$
  - conformal vacuum: thermal bath
- Challenge to Trans-Planckian Challenge:
  - Milne universe is inflating
  - $T_{\mu\nu}$ either zero or thermal bath
  - No contributions to $T_{\mu\nu}$ from smallest scales
  - $\Rightarrow$ No Trans-Planckian effect!?

Conclusions

- Scanning Inflation
  - generalized parametrization of inflationary history
  - many degrees of freedom
  - pandorra's box
  - simulated data sets
  - (implicit) priors matter
  - adjusting priors
  - future experiments can beat priors for r not too small
- Roulette Inflation
  - axionic direction
  - lots of efolds
  - initial conditions
  - low tensors
  - stochastic regime with self reproduction
- Trans-Planckian effects in the Milne Universe
  - Minkowski space reparametrized as FLRW-type
  - No trans-Planckian effects for Minkowski observer $\Leftrightarrow$ Milne observer