Inflation then and now: How can we reliably determine whether dark energy is given by a cosmological constant or whether the power spectrum of scalar fluctuations is exactly scale invariant?

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G. Starkman, R. Trotta, PMV, `arXiv:0811.2415`
G. Starkman, M. March, R. Trotta, PMV, `arXiv:0901.XXXX`

The Unknown

As we know,
There are known knowns.
There are things we know we know.
We also know
There are known unknowns.
That is to say
We know there are some things
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But there are also unknown unknowns,
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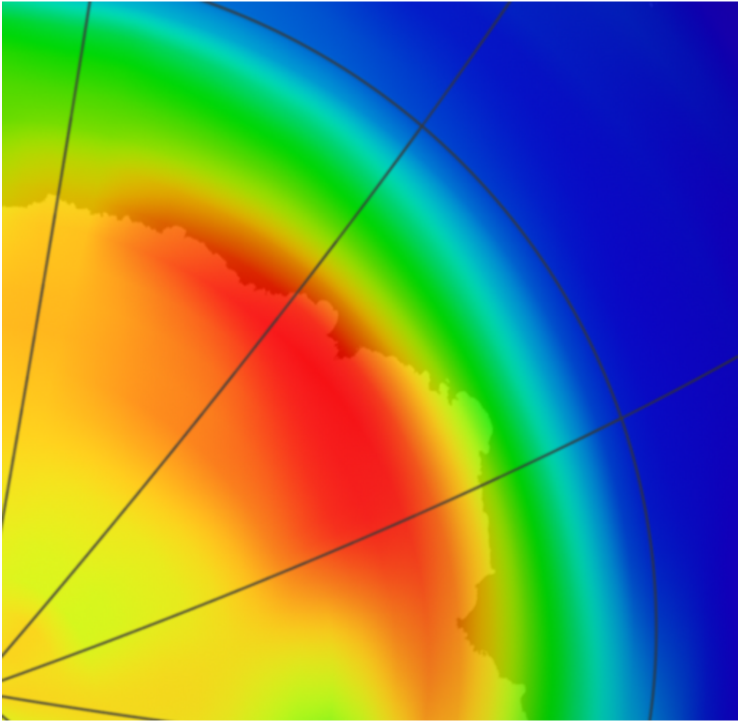
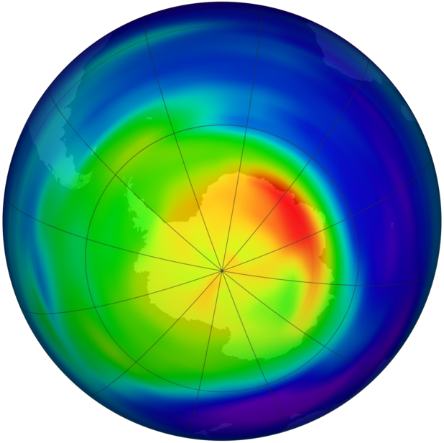
(D. Rumsfeld)

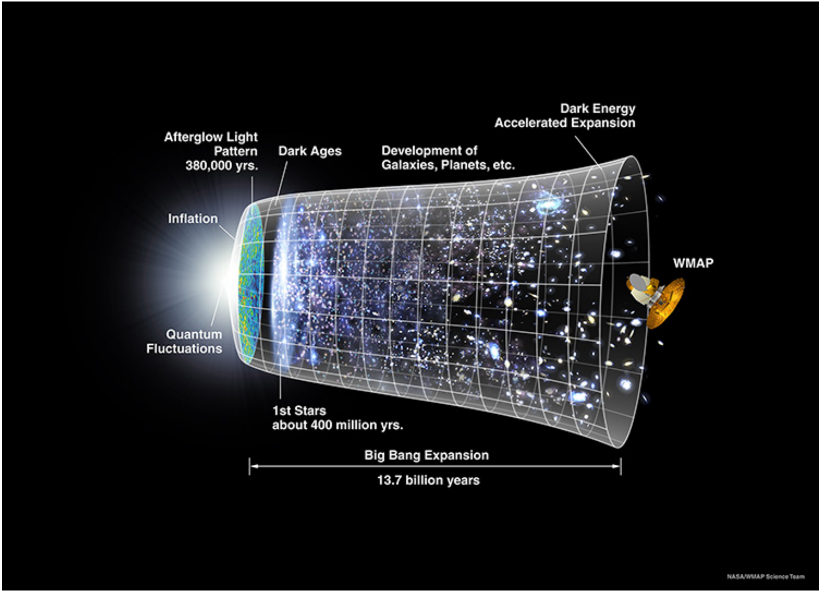




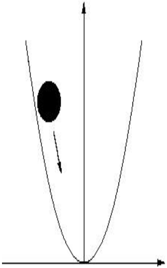
(Microwave Limb Sounder, NASA)

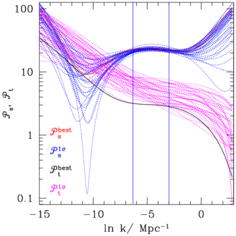
Outline

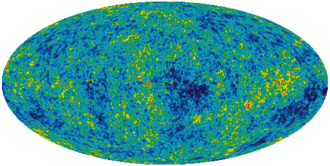
Early and late time inflation



(Early time) inflation

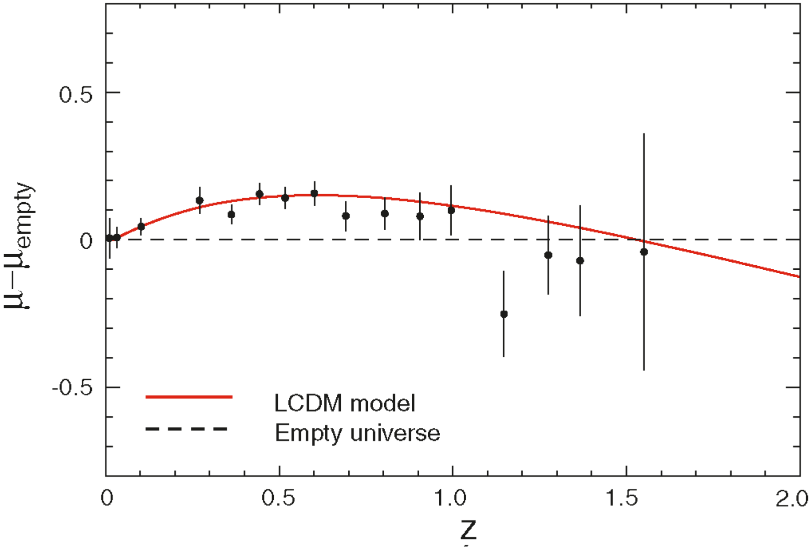






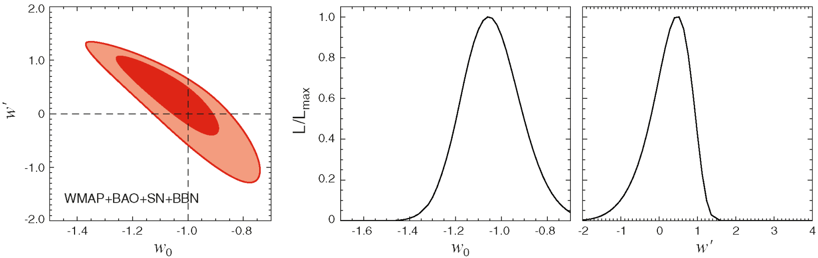
$$\mathcal{P}_s = \frac{k^3}{2\pi^2}\langle \mathcal{R}\mathcal{R} \rangle = \frac{1}{8\pi^2}\frac{H^2}{\epsilon {M_{\mathtt{pl}}}^2} = A_s k^{n_s - 1}$$

Late time inflation/ Dark energy



(WMAP5, Dunkley et al.)

Late time inflation/ Dark energy

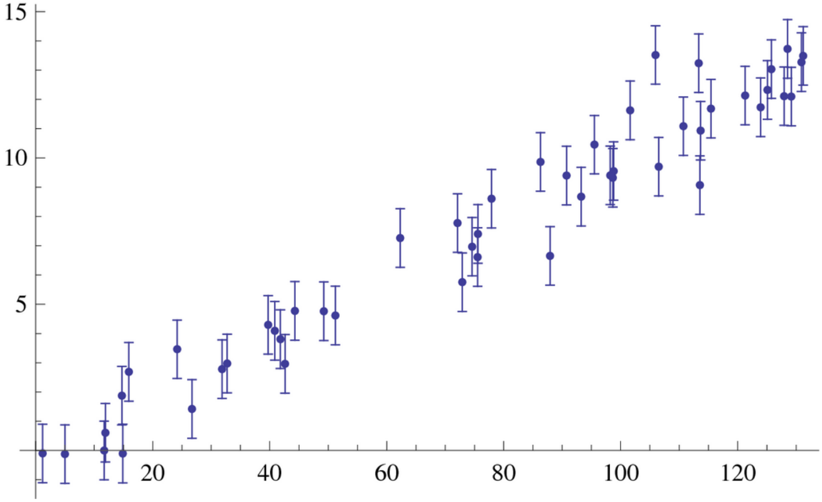


(WMAP5, Komatsu et al.)

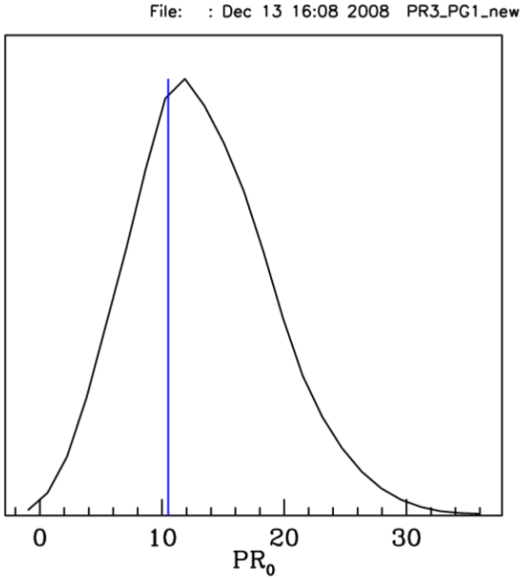
Key problems

- exactly flat power spectrum from inflation?
- dark energy is cosmological constant?

How to detect features?



- visual inspection
- χ^2/dof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Properties of hypothetical criterion for degree of belief

- correct belief
- wrongful belief
- correct doubt
- wrongful doubt

Bayesian model selection

$$p(\mathcal{M}|d) = \frac{p(d|\mathcal{M})p(\mathcal{M})}{p(d)},$$

- evidence $p(d|\mathcal{M}) = \int d\theta \, p(d|\theta, \mathcal{M})p(\theta)$
- penalty for prior volume $\int d\theta \, p(\theta)$
- normalization constant $p(d) = \sum_i p(d|\mathcal{M}_i)p(\mathcal{M}_i)$ (hard to compute, normally ignored)
- Bayes factor $B_{01} = \frac{p(d|\mathcal{M}_0)}{p(d|\mathcal{M}_1)}$

Doubt

$$\mathcal{D} \equiv p(\mathcal{X}|d) = \frac{p(d|\mathcal{X})p(\mathcal{X})}{p(d)}, \mathcal{R} = \frac{\mathcal{D}}{p(\mathcal{X})}$$

- unknown model $\mathcal{X}$
- estimate evidence $p(d|\mathcal{X})$
- normalization constant computable
  $p(d) = p(d|\mathcal{M})p(\mathcal{M}) + p(d|\mathcal{X})p(\mathcal{X})$
- $\ln \mathcal{R} > 0$: model should be doubted

Estimating Evidence - Bayesian Information Criterion

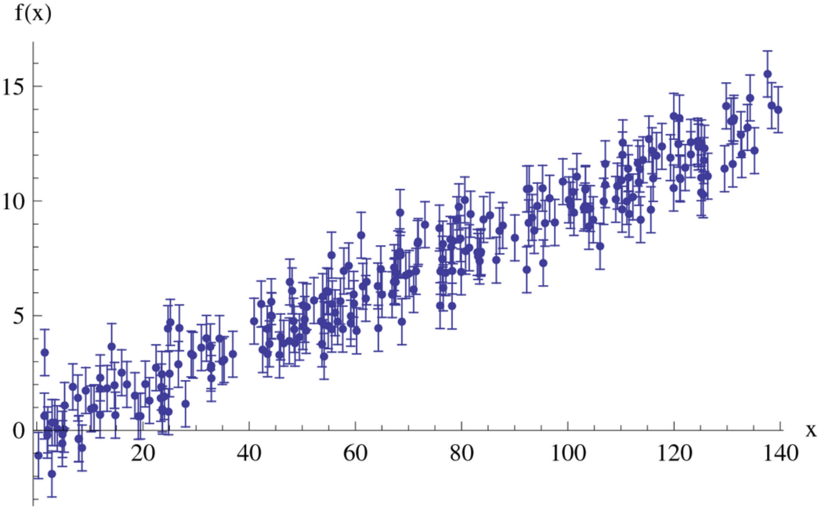
$$p(d|\mathcal{X}) = e^{-\frac{1}{2}\mathrm{BIC}} = \mathcal{L}_{\mathrm{max}} n^{-\frac{1}{2}k}$$

- number of data points n
- number of parameters of the model k
- $\mathcal{L}_{\mathrm{max}} = e^{-\frac{n-k}{2}\frac{\chi^2}{\mathrm{dof}}}$

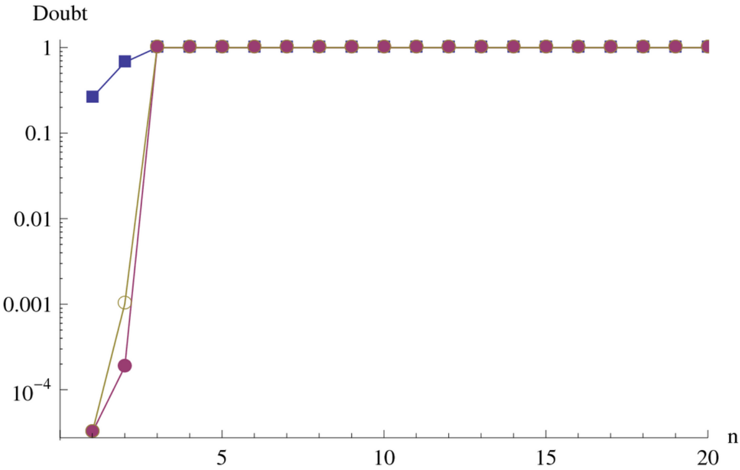
$$\Rightarrow p(d) = p(d|\mathcal{M})p(\mathcal{M}) + p(d|\mathcal{X})p(\mathcal{X})$$

Example

$$f(x) = \frac{1}{10}x$$

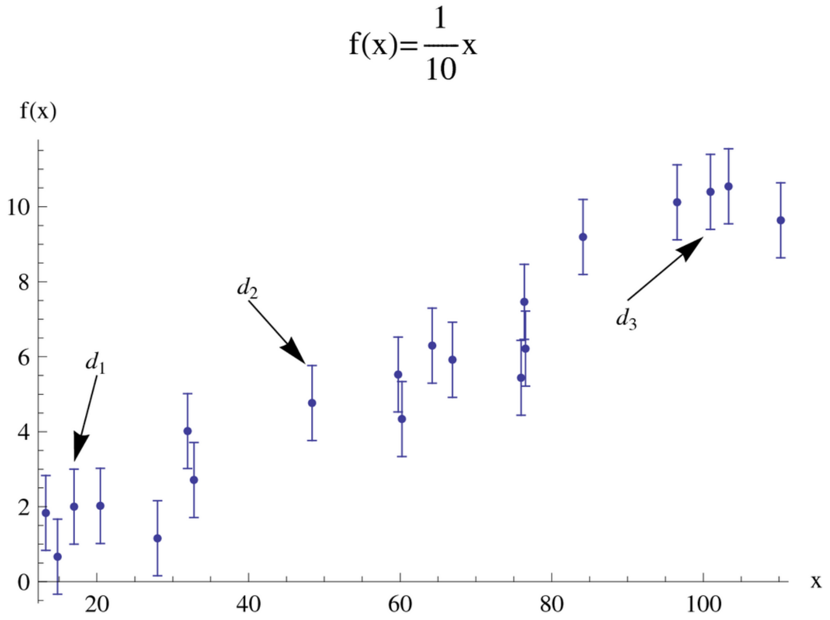


Using the wrong model

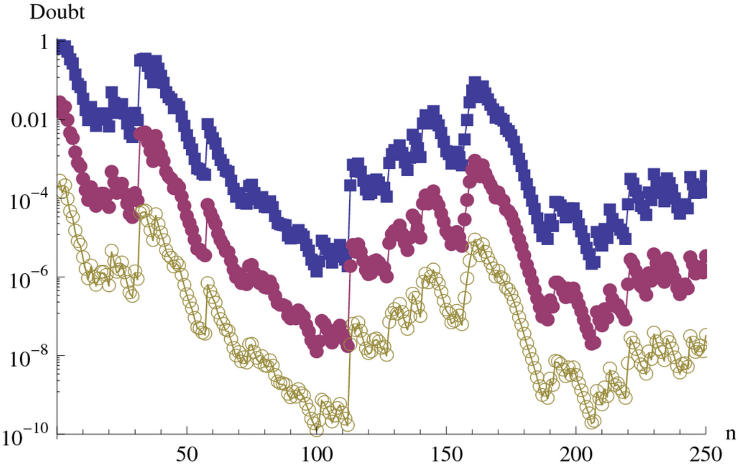


- $p(\mathcal{X}) = 10^{-1}$ (solid blue box)
- $p(\mathcal{X}) = 10^{-5}$ (solid purple circle)

Using the wrong model

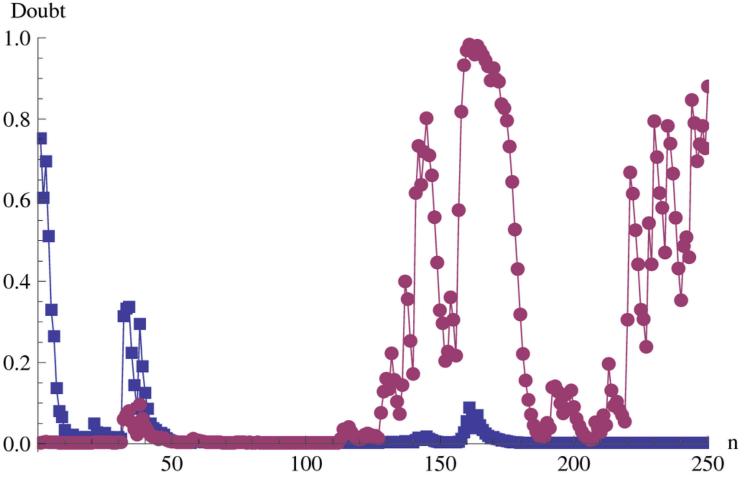


Using the correct model



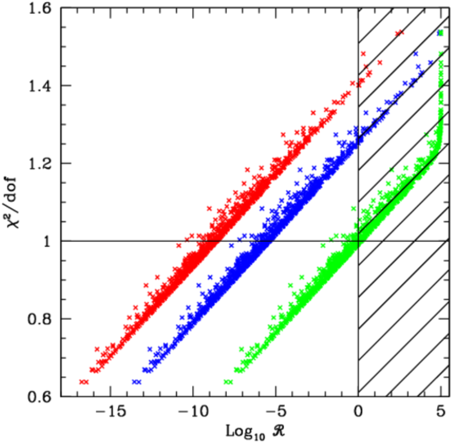
- $p(\mathcal{X}) = 10^{-1}$ (solid blue box)
- $p(\mathcal{X}) = 10^{-3}$ (solid purple circle)
- $p(\mathcal{X}) = 10^{-5}$ (open olive circle)

Estimating $\mathcal{L}_{\max}$



- $\chi^2/\text{dof}=1$: purple circle, too restrictive
- want $\widehat{\mathcal{L}}_{\max} < \mathcal{L}^{\text{true}}_{\max}$ in e.g. 95% of cases
- correct estimator $\widehat{\mathcal{L}}_{\max}$: blue boxes

Performance over 1000 realizations of the data



- green: 50% wrongful doubt
- blue: 5% wrongful doubt
- red: 1% wrongful doubt

Is dark energy the cosmological constant?

- computational cost of posterior
  $p(d|M) = \int d\theta\, p(d|\theta, M) p(\theta)$
- proper calibration
- compute the (increase in) doubt for $w = -1$ and find

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$$\mathcal{D} = ?$$

Conclusions

- how to compute the degree of belief in given model
- weaknesses of model comparison
- (increase in) doubt $\mathcal{R}, (\mathcal{D})$: single number to quantify degree of (dis)belief
- works well for linear toy model
- currently applying to SN-data